The Short-Run Security-Constrained Economic Dispatch

Olga Galland

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THE SHORT-RUN SECURITY-CONSTRAINED ECONOMIC DISPATCH

by

OLGA GALLAND

Supervised and Examined by

Dr. Mohammad R. Hesamzadeh

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Abstract

In liberalized electricity markets, the outputs of controllable units (both generators and demands) must be defined at regular time intervals ("dispatch intervals"). Nowadays, balancing services are procured and dispatched not in the most efficient way partly due to long dispatch intervals. The dispatch interval in most European countries is one hour. The shortest dispatch interval is five minutes and is used in the Australian National Electricity Market (NEM). During the dispatch interval, demand and wind power capacity fluctuates a lot. To keep the supply-demand balance in the system, some generators participate in frequency control. This action increases the system operation cost.

By reducing the dispatch interval to short periods of time over which physical limits of the power system are fully respected, balance services could be dispatched in a more efficient way. This improves the overall economic efficiency of the system.

This work derives the mathematical model for short-run economic dispatch. For this modeling, three stages are considered: (1) initial steady state in which the system might be exposed to a change, (2) the transition period which models the transition cost after the change happened and before the system goes to another steady state equilibrium, and finally (3) the final steady state equilibrium which models the system cost when the change in the system has been handled by the flexible generating
units.

These three stages are modeled in a single optimization problem. The developed optimization problem is a linear programming problem. The developed formulation for the short-run economic dispatch is modeled in GAMS platform. Two applications of the proposed model are discussed: (1) power system security, and (2) real-time balancing market. In the first application, analysis of the optimal dispatch of nodal operating reserves to provide sufficient flexibility to survive a set of credible contingencies is performed. In the second application, an algorithm for the dispatch of balancing services in the real-time balancing market is proposed. These two applications of the proposed short-run economic dispatch are tested on a simple six-bus example system and IEEE twenty-four-bus example system. The optimal dispatch is found and conclusions are drawn. The numerical results of the proposed model show promising results.
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Nomenclature

Indexes
\( i \) Generating unit,
\( n \) Bus,
\( l \) Transmission line,
\( t \) Time period of optimization,
\( k \) Possible contingency,
\( NP \) Current time interval.

Inputs
\( ED_n \) Estimated load size in bus \( n \),
\( \bar{G}_i \) Available generation capacity of generation unit \( i \),
\( \bar{P}_l \) Transmission capacity of line \( l \),

Model parameters
\( T \) Time period,
\( \Delta t \) Dispatch interval,
\( Ng \) Total number of generating units,
\( Nn \) Total number of buses,
\( Nl \) Total number of transmission lines,
$N_t$ Total number of periods of optimization,
$N_k$ Total number of possible contingencies,
$N_P$ Number of time intervals,
$N_h$ Hydro power plants,
$D_n$ Load size in bus $n$ before contingency $k$ occurs,
$D_{n,k}$ Load size in bus $n$, during contingency $k$,
$RR_i$ Ramp rate of generating unit $i$,
$c_i$ Production cost of generating unit $i$,
$\Upsilon$ Generation connection matrix,
$B_l$ Susceptance of transmission line $m$,
$H$ PTDF matrix,
$\Phi$ Transmission connection matrix,
$\xi$ Fictitious cost for undelivered or over delivered load,
$\Psi$ Connection matrix for used generating units,
$r$ Interest rate,
$p_k$ Probability of contingency,
$p_{nc}$ Probability of no contingency,
$p_0$ Probability of not having any contingency,
$u_1$ Random number to find if contingency occurs,
$u_2$ Random number to find load change for occurred contingency.
$Y_l$ Congested and not congested line $l$,
$YY_{n,n}$ Preliminary price group matrix,
$YYY_{n,n}$ Price group matrix.
Optimization model variables

\( G_i \)  
Dispatch in initial steady state of generating unit \( i \),

\( \hat{G}_{i,t,k} \)  
Transition dispatch of generating unit \( i \), during contingency \( k \),

\( G'_{i,k} \)  
Dispatch in final steady state of generating unit \( i \), during contingency \( k \),

\( P_l \)  
Optimal power flow in initial steady state in line \( l \),

\( \hat{P}_{l,t,k} \)  
Optimal power flow during transition period in line \( l \), during contingency \( k \),

\( P'_{l,k} \)  
Optimal power flow in final steady state in line \( l \), during contingency \( k \),

\( DC \)  
Dispatch cost of initial steady state,

\( \hat{DC}_{t,k} \)  
Dispatch cost of transition period, during contingency \( k \),

\( DC'_k \)  
Dispatch cost of final steady state, during contingency \( k \),

\( LL_n \)  
Lost load in initial steady state in bus \( n \),

\( \hat{LL}_{n,t,k} \)  
Lost load during transition period in bus \( n \), during contingency \( k \),

\( LL'_{n,k} \)  
Lost load in final steady state in bus \( n \), during contingency \( k \),

\( U_n \)  
Undelivered load in initial steady state in bus \( n \),

\( \hat{U}_{n,t,k} \)  
Undelivered load during transition period in bus \( n \), during contingency \( k \),

\( U'_{n,k} \)  
Undelivered load in final steady state in bus \( n \), during contingency \( k \),

\( O_i \)  
Over produced energy in initial steady state by generating unit \( i \),

\( \hat{O}_{i,t,k} \)  
Over produced energy during transition period by generating unit \( i \), during contingency \( k \),

\( O'_{i,k} \)  
Over produced energy in final steady state by generating unit \( i \), during contingency \( k \).

Outputs

\( G_i \)  
Optimal dispatch of generating unit \( i \),

\( P_l \)  
Optimal power flow of transmission line \( l \),
$DC$  Dispatch cost,
$U_n$  Undelivered load in bus $n$,
$O_i$  Overproduced energy in generating unit $i$,
$\lambda_n$  Price in bus $n$. 
Chapter 1

Introduction

1.1 Background

The electric power market in Europe is recently on the way to the deregulated market. The main idea is to eliminate monopoly in the electric power industry and instead have competition between several players. The driving force is both economic and production efficiency as deregulation aims to decrease operation cost and consequently electricity price as well as to improve usage of the resources, providing better public services. The power system in a liberalized electricity market is operated by an independent system operator. He is responsible for a reliable and safe operation of a network [3].

The process of selling electricity in a liberalized electricity market can be described as few trading periods (figure 1.1): the ahead trading, the real-time trading, the post trading [4].

All trading which occurred before the actual trading period are called the ahead trading. The players of electricity market make short- or/and long-term agreements. There are few types of agreements in the ahead trading: bilateral trading and financial
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trading. Bilateral trading covers the agreements between two players (consumers and producers) and it should be reported to the system operator unlike agreements in the financial trading. In a spot market the traded quantity is determined by submitted players’ selling and buying bids (figure 1.2) in every trading period (one hour in most countries in Europe) [4] and [5].

Trading occurring during an actual trading period are called real-time trading. The most important player here is system operator. One of the way to design real-time trading is real-time balancing market. The producers/consumers themselves decide their levels of production/consumption, however if it is needed the system operator can ask the player to change its production/consumption [4].

After the trading period is finished, post trading starts, and the system operator compares actual players’ production/consumption with the announced ones in the ahead market. Depending on if the imbalance is negative or positive, players buy/sell imbalanced power to/from the system operator [4].
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More detailed information about electricity market structure can be found in references [4], [5] and [6].

The economic aspect of power system operation becomes a priority as producers are interested in increasing their profit. However, security always has been and remains an important aspect of power system operation. Therefore it is important to find an optimal economic dispatch of all controllable generating units of the power system at regular time intervals (dispatch interval) to insure secure operation of the system. This thesis proposes the model of short-run economic dispatch to consider power system security in an economically efficient manner. In this thesis two applications of this model are discussed. The first application is power system security and the second one is real-time balancing market. The first application is focused on probabilistic security analysis with low frequency contingencies such as loss of generator or load. The power system must be continuously in supply-demand balance through the most
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economically efficient operation. For secure operation of the system, contingencies are managed through a combination of preventive and corrective actions. Corrective actions are actions by the system operator after a contingency occurs (increase or decrease generation level in the power system). Preventive actions are actions by the system operator before contingency occurs in order to prepare for possible contingency (holding back some generation capacity). The second application is focused on optimal dispatch of generating units in the real-time market. In real-time trading all players follow the plan submitted to system operator in ahead trading. However, actual demand as well as wind production deviates from the estimated ones due to the forecast errors. One of the ways to organize the real-time trading is to obligate players to follow the dispatch instructions given by the system operator each dispatch interval. The dispatch interval in most European countries is one hour. During this interval, demand and wind power capacity fluctuate a lot, therefore balance between generation and demand is kept by generating units participating in frequency controls. More information about frequency control can be found in [6], [7] and [8]. The shortest dispatch interval is five minutes and is used in the Australian National Electricity Market (NEM). Furthermore, the dispatch process does not consider all physical limits of the system. By ignoring transmission constraints in the procurement and dispatch of the balance services, it is necessary to leave large reserves in transmission capacities in order to cover possible contingency. Moreover in existing models ramp-rates of the generating units of the power system are not taken into account which brings unrealistic dispatch. So, dispatch of balancing services is imperfect and inefficient. By reducing the dispatch interval to the time over which physical limits of the power system will be respected, balance services could be dispatched in a
more efficient way. Therefore, finding an optimal dispatch would maximize economic success.

There are several papers which are looking at security-constrained economic dispatch or unit commitment formulation, and the optimal procurement of ancillary services or/and reserves. Different formulation of the security-constrained unit commitment is performed in references [9] and [10]. The main assumption in these papers is that contingency has already occurred. In reference [11] the author considers the credibility of the contingency and expected cost of the corrective and preventive actions in power system security. The impact of the preventive actions on the corrective ones is shown in [12]. Authors also describe the benefits of the short dispatch interval. Reference [13] presents new necessary conditions for calculation of the optimal dispatch and prices in a single period, and proposes the methodology for solving them. Authors of reference [14] solve optimal dispatch problem as a mix integer linear problem by minimizing scheduling cost. Reference [15] proposes the model for a power system including renewable energy resources, which depends on climate data. In reference [16], bi-level economic dispatch model is proposed for spot and reserve markets. The upper level determines energy and reserve schedules subjected to normal constrains and in the lower level economic dispatch of reserves is checked for possible contingencies. A hybrid dispatch method for solving ancillary dispatch problems is presented in reference [17]. The model includes both sequential and joint dispatch methods. Reference [18] performs power flow model by minimizing the summation of production cost and ancillary services costs. There is no dispatch model when contingency occurs. In references [19], [20] and [21] methodology for the determination of energy and reserves services in simultaneous optimization is presented. The
need for ancillary services is found through consideration of possible contingencies. In reference [22], utilization of optimal active and reactive power flow algorithms for re-dispatching in an ancillary services market is shown. In [23] different study cases are performed to study the effects of different trading arrangements, planning horizon, pricing imbalances and flexibility of demand on players of an electricity market. In [24], authors look at impact of the stochastic nature of the wind power production on a power system. The paper proposes a probabilistic model of a power system with integration of wind power for the real time balancing market. Most of these papers do not consider ramp rates of generating units and transmission lines limits and are based on usual dispatch intervals.

This thesis proposes a model of short-run economic dispatch under consideration of system physical limits. The main outputs of the model are optimal generation dispatch, dispatch cost and price in each bus. Prices in the first application are based on nodal pricing. Prices in the second application are calculated based on area pricing [25], [26], [27] and [28]. The principles of area pricing are described in Appendix B.

1.2 Problem Definition

In this work a mathematical model for the short-run economic dispatch is developed. The developed mathematics is implemented in GAMS and MATLAB platforms and is then tested on the academic case studies. The results of this implementation are discussed for further application to the industrial environment. Also the dispatching results of the short-run economic dispatch are compared to the existing dispatch method currently used in electricity markets. The results of this research work will
be presented as a journal paper.

1.3 Objective

The objective of this project is to propose the model of short-run economic dispatch that respects physical limits of the system and minimizes the total dispatch cost. The project also aims to provide two applications of the model: power system security and real-time balancing market. This thesis project investigates the impact of short dispatch interval on system dispatch cost and system price.

1.4 Overview of the Report

In Chapter 2, the derivation of the mathematical model used for short-run economic dispatch is presented. Then, the proposed model is formulated and explained in Chapter 3. Chapters 4 and 5 describe two studied cases and present the results. Chapter 6 concludes and outlines future work.
Chapter 2

Derivation of Mathematical Model

The modeling framework is shown in figure 2.1. The initial condition is that the system is in a steady state equilibrium, $s$. In each period there is probability $p_k$ that contingency $k$ occurs and probability $p_0 = 1 - p_k$ that it does not occur. If contingency does not occur the system remains in steady state. Otherwise, to keep balance in the system, the system moves to the transition to a new steady state equilibrium, where the generation is changing each time interval until optimal dispatch cost is reached. Thereby the system comes to a new steady state equilibrium, $s'$. We are assuming that no contingency occurs while system is not in steady state equilibrium.

Nowadays it is important not only to keep balance of the system but also do it with the lowest possible cost. Mathematically this problem can be described as summation of dispatch costs over all considered time intervals. Let $DC(s)$ be the dispatch cost of the system in initial steady state, $s$, and $DC(s')$ be the dispatch cost of the system in final steady state, $s'$. Let’s also assume that transition period from $s$ to $s'$ takes at most $N_t$ periods. Then we can find the additional dispatch cost (cost of transition period), $AC(s \rightarrow s')$: 
Figure 2.1: The modeling framework

\[
AC(s \rightarrow s') = \sum_{t=1}^{N_t} \frac{(DC(s_t) - DC(s'))}{(1 + r)^t}
\]  

(2.1)

Where \( s_t \) is the state of the power system in time period \( t \) after the contingency occurred and \( r \) is the interest rate per period.

The present value of the future stream of dispatch costs is:

\[
AC(s \rightarrow s') + \frac{DC(s')}{(1 + r)} + \frac{DC(s')}{(1 + r)^2} + ... \\
= AC(s \rightarrow s') + \frac{DC(s')}{r}
\]  

(2.2)
Thus, the present value of the expected dispatch cost of the power system is:

\[ PV(DC) = pnc \cdot DC(s) + p \cdot \left( AC(s \rightarrow s') + \frac{DC(s')}{r} \right) \]

\[ + \frac{(1-p)}{(1+r)} \cdot \left[ pnc \cdot DC(s) + p \cdot \left( AC(s \rightarrow s') + \frac{DC(s')}{r} \right) \right] \]

\[ + \frac{(1-p)^2}{(1+r)^2} \cdot \left[ pnc \cdot DC(s) + p \cdot \left( AC(s \rightarrow s') + \frac{DC(s')}{r} \right) \right] \]

\[ + \ldots \]

\[ = \frac{(1+p)}{(1+r)} \cdot \left[ pnc \cdot DC(s) + p \cdot \left( AC(s \rightarrow s') + \frac{DC(s')}{r} \right) \right] \] (2.3)

As seen from equation 2.3, if probability of contingency is zero, the present value is equal to the dispatch cost of the initial steady state equilibrium. Otherwise present value is equal to the minimum possible sum of the initial dispatch cost and expected cost of adjusting to a new steady state.
Chapter 3

Formulation of Proposed Model

The optimal dispatch model is executed in the GAMS platform [29] and [30]. The input data is stored in Microsoft Excel and converted in readable by GAMS gdx files by MATLAB. Outputs of GAMS simulation are exported to MATLAB to exploit the results. The GAMS/MATLAB interface is described in [31]. The model was done for a general system and can therefore easily be adapted for any other power system. This work is focused on two applications of the above described mathematical model (see Chapter 2). The first application is focused on probabilistic security analysis with low frequency, large contingencies to the power system, like loss of generating unit or load. The second application is in real-time balancing market to deal with high frequency contingencies such as deviation of the load from the estimated one.

3.1 Assumptions

For simplicity of the model, the following assumptions were made:

- Lines in the system are lossless, therefore consumption of the system is equal to the generation;
3.2. MODEL DESCRIPTION

- All lines are always available;
- All generators are always available;
- Minimum time interval for load change is one minute, i.e. load is constant for one minute;
- No contingency occurs while system is not in steady state equilibrium;
- Load is not price sensitive, demand is independent of the state of the system,
- Marginal pricing, the price is set by the maximum production cost;
- Perfect competition, none of the owners use market power.

3.2 Model Description

3.2.1 Probabilistic Security Analysis

This section of the thesis shows the possibility for secure operation of the system for low frequency contingency using the proposed mathematical model.

The optimal dispatch of the system is found by running the optimization problem for initial steady state, transition period and final steady state. The objective function of this optimization problem is aimed to find an optimal solution for the system during possible contingencies $k$ by minimizing total production cost:
3.2. MODEL DESCRIPTION

\begin{align*}
\text{minimize} & \quad G_i, \tilde{G}_{i,t,k}, G_{i,k}', LL_n, LL_{n,t,k}, LL_{n,k}' \\
& \sum_{k=1}^{N_k} \left[ \left( 1 - p_k \right) \cdot \left( \sum_{i=1}^{N_g} c_i \cdot G_i + \sum_{n=1}^{N_n} \xi \cdot LL_n \right) \\
& + \sum_{t=1}^{N_t} \left( \frac{p_k}{(1 + r)^t} \cdot \left( \sum_{i=1}^{N_g} c_i \cdot \tilde{G}_{i,t,k} + \sum_{n=1}^{N_n} \xi \cdot \hat{LL}_{n,t,k} \right) \right) \\
& + \frac{p_k}{(1 + r)^{N_t} \cdot r} \cdot \left( \sum_{i=1}^{N_g} c_i \cdot G'_{i,k} + \sum_{n=1}^{N_n} \xi \cdot LL'_{n,k} \right) \right] + r \right] \right]
\end{align*}

\text{(3.1)}

Subject to

- Energy balance constrains:
  \begin{align*}
  & \sum_{i=1}^{N_g} (G_i - O_i) \cdot \Upsilon_{i,n} + \sum_{l=1}^{N_l} P_l \cdot \Phi_{l,n} + U_n - D_n = 0 \\
  & \sum_{i=1}^{N_g} \left( \tilde{G}_{i,t,k} - \hat{O}_{i,t,k} \right) \cdot \Upsilon_{i,n} + \sum_{l=1}^{N_l} \tilde{P}_{l,t,k} \cdot \Phi_{l,n} + \hat{U}_{n,t,k} - D_{n,k} = 0 \\
  & \sum_{i=1}^{N_g} (G'_{i,k} - O'_{i,k}) \cdot \Upsilon_{i,n} + \sum_{l=1}^{N_l} P'_{l,k} \cdot \Phi_{l,n} + U'_{n,k} - D_{n,k} = 0
\end{align*}

\text{(3.2)}

- Transmission flow constrains:
  \begin{align*}
  & \sum_{n=1}^{N_n} \left[ H_{l,n} \cdot \left( \sum_{i=1}^{N_g} (G_i - O_i) \cdot \Upsilon_{i,n} - (D_n - U_n) \right) \right] = P_l \\
  & \sum_{n=1}^{N_n} \left[ H_{l,n} \cdot \left( \sum_{i=1}^{N_g} (\tilde{G}_{i,t,k} - \hat{O}_{i,t,k}) \cdot \Upsilon_{i,n} - (D_{n,k} - \hat{U}_{n,t,k}) \right) \right] = \tilde{P}_{l,t,k} \\
  & \sum_{n=1}^{N_n} \left[ H_{l,n} \cdot \left( \sum_{i=1}^{N_g} (G'_{i,k} - O'_{i,k}) \cdot \Upsilon_{i,n} - (D_n - U'_{n,k}) \right) \right] = P'_{l,k}
\end{align*}

\text{(3.3)}

- Lost load limits constrains:
  \begin{align*}
  LL_n = U_n + \sum_{i=1}^{N_g} O_i \cdot \Upsilon_{i,n}
\end{align*}
3.2. MODEL DESCRIPTION

\[ \hat{L}L_{n,t,k} = \hat{U}_{n,t,k} + \sum_{i=1}^{N_g} \hat{O}_{i,t,k} \star \Upsilon_{i,n} \]

\[ LL'_{n,k} = U'_{n,k} + \sum_{i=1}^{N_g} O'_{i,k} \star \Upsilon_{i,n} \] (3.4)

- Transmission flow limits constrains,

\[-P_i \leq P_i \leq P_i \]
\[-\bar{P}_i \leq \bar{P}_{i,t,k} \leq \bar{P}_i \]
\[-P_i \leq P'_{i,k} \leq \bar{P}_i \] (3.5)

- Generation limits constrains:

\[ 0 \leq G_i \leq \bar{G}_i \]
\[ 0 \leq \hat{G}_{i,t,k} \leq \bar{G}_i \]
\[ 0 \leq G'_{i,k} \leq \bar{G}_i \] (3.6)

- Ramp-rate limits constrains:

\[ 0 \leq |\hat{G}_{i,t,k} - \hat{G}_{i,t-1,k}| \leq RR_i \]
\[ 0 \leq |G'_{i,k} - \hat{G}_{i,T_o,k}| \leq RR_i \] (3.7)

The optimization problem (3.1 - 3.7) is solved by CPLEX solver in GAMS platform as a linear programming problem. Therefore, Lagrange multipliers can be found to determine the nodal prices:

- Energy balance constrains \( \rightarrow \mu_n, \hat{\mu}_{n,t,k}, \mu'_{n,k} \)
- Transmission flow constrains \( \rightarrow \gamma_n, \hat{\gamma}_{l,t,k}, \gamma'_{l,k} \)
3.2. MODEL DESCRIPTION

- Transmission flow limits constrains → $\nu_n, \hat{\nu}_{t,t,k}, \nu'_{t,k}$ (only for congested transmission lines)

for initial, transient and final states.

The Lagrange function:

- Initial steady state:

$$\Lambda_n = \sum_{k=1}^{N_k} \frac{(1 - p_k) (1 + r)}{(p_k + r)} \left( \sum_{i=1}^{N_g} c_i \cdot \hat{G}_{i,t,k} + \sum_{n=1}^{N_n} \xi \cdot \hat{L}L_n \right)$$

$$- \mu_n \cdot \left( \sum_{i=1}^{N_g} G_i \cdot D_n + \sum_{l=1}^{N_l} P_l \cdot \Phi_{l,n} \right)$$

$$- \sum_{l=1}^{N_l} \left( \gamma_l \cdot \left( P_l - \sum_{n=1}^{N_n} H_{l,n} \cdot \left( \sum_{i=1}^{N_g} G_i \cdot \Upsilon - D_n \right) \right) \right)$$

$$- \sum_{l=1}^{N_l} \left( \nu_l \cdot \left( \bar{P}_l - \sum_{n=1}^{N_n} H_{l,n} \cdot \left( \sum_{i=1}^{N_g} G_i \cdot \Upsilon - D_n \right) \right) \right)$$

- Transient state:

$$\hat{\Lambda}_{n,t,k} = \frac{p_k (1 + r)}{(p_k + r) (1 + r)} \left( \sum_{i=1}^{N_g} \hat{G}_{i,t,k} + \sum_{n=1}^{N_n} \xi \cdot \hat{L}L_{n,t,k} \right)$$

$$- \hat{\mu}_{n,t,k} \cdot \left( \sum_{i=1}^{N_g} \hat{G}_{i,t,k} - D_{n,k} + \sum_{l=1}^{N_l} \hat{P}_{l,t,k} \cdot \Phi_{l,n} \right)$$

$$- \sum_{l=1}^{N_l} \left( \hat{\gamma}_{l,t,k} \cdot \left( \hat{P}_{l,t,k} - \sum_{n=1}^{N_n} H_{l,n} \cdot \left( \sum_{i=1}^{N_g} \hat{G}_{i,t,k} \cdot \Upsilon - D_{n,k} \right) \right) \right)$$

$$- \sum_{l=1}^{N_l} \left( \hat{\nu}_{l,t,k} \cdot \left( \hat{P}_l - \sum_{n=1}^{N_n} H_{l,n} \cdot \left( \sum_{i=1}^{N_g} \hat{G}_{i,t,k} \cdot \Upsilon - D_{n,k} \right) \right) \right)$$

- Final steady state:
\[ \Lambda'_{n,k} = \frac{p_k (1 + r)}{(p_k + r) (1 + r) N_t r} \left( \sum_{i=1}^{N_g} c_i \cdot G'_{i,k} + \sum_{n=1}^{N_n} \xi \cdot LL'_{n,k} \right) \]

\[ - \mu'_{n,k} \cdot \left( \sum_{i=1}^{N_g} G'_{i,k} - D_{n,k} + \sum_{l=1}^{N_l} P'_{l,k} \cdot \Phi_{l,n} \right) \]

\[ - \sum_{l=1}^{N_l} \left( \gamma'_{l,k} \cdot \left( P'_{l,k} - \sum_{n=1}^{N_n} H_{l,n} \cdot \left( \sum_{i=1}^{N_g} G'_{i,t,k} \cdot \Upsilon - D_{n,k} \right) \right) \right) \]

\[ - \sum_{l=1}^{N_l} \left( \nu'_{l,k} \cdot \left( \tilde{P}_l - \sum_{n=1}^{N_n} H_{l,n} \cdot \left( \sum_{i=1}^{N_g} G'_{i,t,k} \cdot \Upsilon - D_{n,k} \right) \right) \right) \]  

(3.8)

then

- Initial steady state:

\[ \frac{\partial \Lambda_n}{\partial G_n} = \sum_{k=1}^{N_k} \frac{(1 - p_k) (1 + r)}{(p_k + r)} \lambda_n - \mu_n + \sum_{l=1}^{N_l} \gamma_l \cdot H_{l,n} + \sum_{l=1}^{N_l} \nu_l \cdot H_{l,n} = 0 \]

- Transient state:

\[ \frac{\partial \hat{\Lambda}_{n,t,k}}{\partial \hat{G}_{n,t,k}} = \frac{p_k (1 + r)}{(p_k + r) (1 + r)^2} \hat{\lambda}_{n,t,k} - \hat{\mu}_{n,t,k} + \sum_{l=1}^{N_l} \hat{\gamma}_{l,t,k} \cdot H_{l,n} + \sum_{l=1}^{N_l} \hat{\nu}_{l,t,k} \cdot H_{l,n} = 0 \]

- Final steady state:

\[ \frac{\partial \Lambda'_{n,k}}{\partial G'_{n,k}} = \frac{p_k (1 + r)}{(p_k + r) (1 + r) N_t r} \lambda'_{n,k} - \mu'_{n,k} + \sum_{l=1}^{N_l} \gamma'_{l,k} \cdot H_{l,n} + \sum_{l=1}^{N_l} \nu'_{l,k} \cdot H_{l,n} = 0 \]  

(3.9)

Therefore nodal prices can be defined as:
### 3.2. Model Description

- **Initial steady state:**

\[
\lambda_n = \frac{1}{\sum_{k=1}^{N_k} \frac{(1-p_k)(1+r)}{p_k+r}} \left( \mu_n - \sum_{l=1}^{N_l} \gamma_l \cdot H_{l,n} - \sum_{l=1}^{N_l} \nu_l \cdot H_{l,n} \right)
\]

- **Transient state:**

\[
\hat{\lambda}_{n,t,k} = \left( \frac{p_k + r}(1 + r) \right)^t \frac{1}{p_k(1+r)} \left( \hat{\mu}_{n,t,k} - \sum_{l=1}^{N_l} \hat{\gamma}_{l,t,k} \cdot H_{l,n} - \sum_{l=1}^{N_l} \hat{\nu}_{l,t,k} \cdot H_{l,n} \right)
\]

- **Final steady state:**

\[
\lambda'_{n,k} = \left( \frac{p_k + r}(1 + r) \right)^{N_t} r \left( \mu'_{n,k} - \sum_{l=1}^{N_l} \gamma'_{l,k} \cdot H_{l,n} - \sum_{l=1}^{N_l} \nu'_{l,k} \cdot H_{l,n} \right)
\]

The nodal prices for each state have three components: (1) system marginal price, (2) marginal congestion component and (3) marginal security component.

#### 3.2.2 Real-Time Balancing Market

This part of the thesis is focused on high frequency contingencies in the power system, such as deviations of load in the system from the estimated one. The assumption is that all other disturbances (like loss of generator or transmission line) occur on sufficiently long time scales compared to the time scale of regarded contingencies.

The proposed model is operated as shown in figure 3.1. The assumption is that no contingency appears while the system does not come to the new steady state equilibrium. Therefore the input data for the optimization at time 2 minutes is equal to output of the previous time period, i.e. 1 minute. The optimal dispatch is done by running optimization problem for initial steady state, transition period and final
steady state. For better understanding of the model its algorithm is described below.

Figure 3.2 presents the algorithm of the proposed model. The algorithm consists of two parts: first find optimal dispatch interval for first contingency, second find optimal dispatch for the rest of the time intervals. The difference is that for the first contingency initial steady state is assumed to be unknown unlike for the following contingencies.

As seen from figure 3.2, first of all the time period $T$ with dispatch interval $\Delta t$ should be stated to model the optimal dispatch for the studied system. So, the number of studied intervals is:

$$NP = \frac{T}{\Delta t}$$  \hspace{1cm} (3.11)
3.2. MODEL DESCRIPTION

Figure 3.2: Block diagram of the proposed model
The next step is to import input data from gdx files, previously created in Mat-
Lab.
The first contingency can then be modeled and the first part of algorithm starts. This
is done in the "Contingency generator" block. The principles of operation are shown
in figure 3.3. Random number $u_2$ is generated to find load changes in each bus from
the estimated load. Therefore:

$$D_n = ED_n + u_2$$  \hspace{1cm} (3.12)

Assuming that the determined load should be

$$0.2 \times ED_n \leq D_n \leq 1.8 \times ED_n$$  \hspace{1cm} (3.13)

And if the load is out of the set boundaries

$$D_n = D_n - 1.5 \times u_2$$  \hspace{1cm} (3.14)
After contingency occurred, the system should still keep balance, i.e. generation should be equal to demand. Therefore the generation in the generating units has to be changed. The aim of this model is to find the optimal dispatch for the studied system. The operation of the system can therefore be described as an optimization problem. The optimization problem is the same as in section 3.2.1, (3.1)-(3.7), except that only one possible contingency is considered at a time, therefore the objective function becomes:
3.2. MODEL DESCRIPTION

\[
\begin{align*}
\text{minimize} & \quad G_i, \hat{G}_{i,t}, G_i', L L_n, L L_{n,t}, L L_n' \\
& \quad (1 - p) \left( \sum_{i=1}^{N_g} c_i \cdot G_i + \sum_{n=1}^{N_n} \xi \cdot L L_n \right) \\
& \quad + \sum_{t=1}^{N_t} \left( \frac{p}{(1 + r)^t} \cdot \left( \sum_{i=1}^{N_g} c_i \cdot \hat{G}_{i,t} + \sum_{n=1}^{N_n} \xi \cdot \hat{L L}_{n,t} \right) \right) \\
& \quad + \frac{p}{(1 + r)^T} \cdot r \cdot \left( \sum_{i=1}^{N_g} c_i \cdot G_i' + \sum_{n=1}^{N_n} \xi \cdot L L_n' \right) \quad (3.15)
\end{align*}
\]

Subject to

- Energy balance constrains:

\[
\begin{align*}
\sum_{i=1}^{N_g} (G_i - O_i) \cdot \Upsilon_{i,n} + \sum_{t=1}^{N_l} P_t \cdot \Phi_{t,n} + U_n - D_n &= 0 \\
\sum_{i=1}^{N_g} (\hat{G}_{i,t} - \hat{O}_{i,t}) \cdot \Upsilon_{i,n} + \sum_{t=1}^{N_l} \hat{P}_{t,n} \cdot \Phi_{t,n} + \hat{U}_{n,t} - D_n &= 0 \\
\sum_{i=1}^{N_g} (G_i' - O_i') \cdot \Upsilon_{i,n} + \sum_{t=1}^{N_l} P_t' \cdot \Phi_{t,n} + U_n' - D_n &= 0 \quad (3.16)
\end{align*}
\]

- Transmission flow constrains:

\[
\begin{align*}
\sum_{n=1}^{N_n} \left[ H_{l,n} \cdot \left( \sum_{i=1}^{N_g} (G_i - O_i) \cdot \Upsilon_{i,n} - (D_n - U_n) \right) \right] &= P_l \\
\sum_{n=1}^{N_n} \left[ H_{l,n} \cdot \left( \sum_{i=1}^{N_g} (\hat{G}_{i,t} - \hat{O}_{i,t}) \cdot \Upsilon_{i,n} - (D_n - \hat{U}_{n,t}) \right) \right] &= \hat{P}_{l,t} \\
\sum_{n=1}^{N_n} \left[ H_{l,n} \cdot \left( \sum_{i=1}^{N_g} (G_i' - O_i') \cdot \Upsilon_{i,n} - (D_n - U_n') \right) \right] &= P_l' \quad (3.17)
\end{align*}
\]

- Lost load limits constrains:

\[
L L_n = U_n + \sum_{i=1}^{N_g} O_i \cdot \Upsilon_{i,n}
\]
\[ \hat{L}L_{n,t} = \hat{U}_{n,t} + \sum_{i=1}^{N_g} \hat{O}_{i,t} \cdot \chi_{i,n} \]
\[ LL'_{n,k} = U'_n + \sum_{i=1}^{N_g} O'_i \cdot \chi_{i,n} \quad (3.18) \]

- Transmission flow limits constrains,
  \[ -\bar{P}_i \leq P_i \leq \bar{P}_i \]
  \[ -\bar{P}_i \leq \hat{P}_{i,t} \leq \bar{P}_i \]
  \[ -\bar{P}_i \leq P'_i \leq \bar{P}_i \quad (3.19) \]

- Generation limits constrains:
  \[ 0 \leq G_i \leq \bar{G}_i \]
  \[ 0 \leq \hat{G}_{i,t} \leq \bar{G}_i \]
  \[ 0 \leq G'_i \leq \bar{G}_i \quad (3.20) \]

- Ramp-rate limits constrains:
  \[ 0 \leq |\hat{G}_{i,t} - \hat{G}_{i,t-1}| \leq RR_i \]
  \[ 0 \leq |G'_i - \hat{G}_{i,To}| \leq RR_i \quad (3.21) \]

The optimization problem in (3.15) - (3.21) is a linear programming problem which is solved using CPLEX solver in GAMS platform.

After the optimal solution was found, the time period at which the system reaches a new steady state equilibrium is found and the results are saved. The number of periods it took to reach a new steady state equilibrium is saved as index. The next step of the model is defining prices in each bus. The algorithm is shown in Fig. 3.4.
Area pricing is used [25], [26], [27], [28] and each bus is assumed to be one area. As each bus is one price area, the price at a bus is equal to the marginal production cost of the most expensive used generator in this bus. Therefore a connection matrix for used generators is created:

$$\Psi_{i,n} = \begin{cases} 
1 & \text{if } \Upsilon_{i,n} \cdot G_i \neq 0 \\
0 & \text{if } \Upsilon_{i,n} \cdot G_i = 0
\end{cases}$$

Then price:

$$\lambda_n = \max (C_i \cdot \Psi_{i,n})$$

Next, vector $Y$ is built, which contains zero and one (zero - line is congested, one - line is not congested). Then it is found if buses are in the same price group by creating matrix $YY$ ($Nn$ by $Nn$):

$$YY_{n,n} = \begin{cases} 
1 & \text{if } \Phi_{l,n} \cdot Y_l = 1 \\
0 & \text{if } \Phi_{l,n} \cdot Y_l = 0
\end{cases}$$

All possible connections need to be checked, therefore:

$$YYY_{n,n} = \begin{cases} 
1 & \text{if } YYY_{n,n} + YY_{n,n} \neq 0 \\
YY_{n,n} & \text{if } YYY_{n,n} + YY_{n,n} = 0
\end{cases}$$

Matrix $YYY$ ($Nn$ by $Nn$) contains zeros and ones. One means that corresponding buses are connected (the same price area) and zero means they are not connected. Now all buses are divided in price groups. The price of each bus is equal to maximum
price in its price group:

$$\lambda_n = \max (\lambda_n \cdot YY n, n)$$  \hspace{1cm} (3.22)

Finally the direction of the flow between groups is checked. If the price of imported power is higher than the price in the considered price group, the price will be set
equal to the price of imported power.
Thereby prices in each node are found for each period of latest optimization.
Then algorithm passes to the next time interval and second part of the algorithm starts:

\[ NP = NP + \text{index} \]  \hspace{1cm} (3.23)

If the next time interval is larger than total number of periods \( \bar{NP} \), results are ex-
ported to the output file and the algorithm exits. If it is lower, the algorithm passes to the next time interval and a new random number \( u_1 \) is generated. Probability of not having any contingency is defined:

\[ p_0 = \prod_{c}^{N_c} (1 - p_n) \]  \hspace{1cm} (3.24)

If the generated random number \( u_1 \) is lower than the probability of not having any contingency, load in each bus will be the same as estimated one. If not, a vector of random number \( u_2 \) is generated and the load is calculated using equation 3.12, checked by 3.13 and if needed corrected by 3.14. Then new optimization is done to keep balance of the system. This optimization is the same as the one described above except that the initial steady state is already known and it is not needed to optimize. Then the results of the latest optimization are saved and prices are calculated. The algorithm repeats the second part of the algorithm until the current number of intervals does not reach the total number of intervals. Finally results for all required periods are reported and the algorithm terminates.

The optimization problem formulated in (3.15) can be used to test if the power system
is flexible enough to cope with variable and unpredictable production of intermittent generating units (such as wind and solar power). By flexibility we mean the ability of the system to increase/decrease its generation the same period as contingency occurred. This ability can be characterized by System Flexibility Index (SFI):

\[
SFI = \frac{1}{\sum_{n=1}^{N_n} (LL_n + \sum_{t=1}^{T} \hat{LL}_{t,n} + LL'_n)}
\]

(3.25)

System with higher \textit{SFI} is more flexible to variable and unpredictable changes in power system.
Chapter 4

Case Studies. System Design

For better understanding, the proposed model was studied on two examples. First, the developed methodology was applied to a simple six-bus system and then the same method was used for twenty-four bus IEEE reliability test system.

4.1 Six-Bus Test System

The six bus test system is shown in figure 4.1.

The used data is fictitious however it is based on the data of the IEEE reliability test system [32]. Generating units data is presented in Table 4.1. Table 4.2 presents the transmission lines data.

Bus 113 is considered as reference bus in our study. Estimated load is taken from table 4.1: Generating units data - 6 bus system.

<table>
<thead>
<tr>
<th>ID</th>
<th>Unit Size, (MW)</th>
<th>Ramp Rate, (MW/Min)</th>
<th>Cost, ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>591</td>
<td>100</td>
<td>48.6</td>
</tr>
<tr>
<td>G2</td>
<td>150</td>
<td>20</td>
<td>5.65</td>
</tr>
<tr>
<td>G3</td>
<td>100</td>
<td>6</td>
<td>0.001</td>
</tr>
<tr>
<td>G4</td>
<td>50</td>
<td>6</td>
<td>12.4</td>
</tr>
<tr>
<td>G5</td>
<td>50</td>
<td>4</td>
<td>11.9</td>
</tr>
</tbody>
</table>
4.1. SIX-BUS TEST SYSTEM

Table 4.2: Transmission lines data - 6 bus system

<table>
<thead>
<tr>
<th>ID</th>
<th>From Bus</th>
<th>To Bus</th>
<th>X, (pu)</th>
<th>Con, (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1221</td>
<td>113</td>
<td>123</td>
<td>0.087</td>
<td>150</td>
</tr>
<tr>
<td>1319</td>
<td>113</td>
<td>119</td>
<td>0.040</td>
<td>150</td>
</tr>
<tr>
<td>2001</td>
<td>113</td>
<td>120</td>
<td>0.040</td>
<td>150</td>
</tr>
<tr>
<td>1321</td>
<td>113</td>
<td>121</td>
<td>0.020</td>
<td>150</td>
</tr>
<tr>
<td>1923</td>
<td>119</td>
<td>123</td>
<td>0.040</td>
<td>150</td>
</tr>
<tr>
<td>2021</td>
<td>120</td>
<td>121</td>
<td>0.090</td>
<td>150</td>
</tr>
<tr>
<td>2022</td>
<td>120</td>
<td>122</td>
<td>0.090</td>
<td>150</td>
</tr>
<tr>
<td>1331</td>
<td>120</td>
<td>123</td>
<td>0.022</td>
<td>150</td>
</tr>
<tr>
<td>2223</td>
<td>122</td>
<td>123</td>
<td>0.090</td>
<td>150</td>
</tr>
</tbody>
</table>

[32] as first fifty hours of hourly load for buses 113, 119 and 120 and is performed in tables 4.3, 4.4 and 4.5. The load is assumed to be constant during one hour.

Tables 4.4 and 4.5 show the daily load in percent of weekly peak and hourly load in
4.1. SIX-BUS TEST SYSTEM

Table 4.3: Load data - 6 bus system

<table>
<thead>
<tr>
<th>Bus</th>
<th>Peak load of one year (MW)</th>
<th>Peak load of week 1 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>113</td>
<td>265</td>
<td>86.2</td>
</tr>
<tr>
<td>119</td>
<td>181</td>
<td>86.2</td>
</tr>
<tr>
<td>120</td>
<td>128</td>
<td>86.2</td>
</tr>
</tbody>
</table>

percent of daily peak respectively.

Table 4.4: Daily load in percent of weekly peak

<table>
<thead>
<tr>
<th>Day</th>
<th>Peak load (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>93</td>
</tr>
<tr>
<td>Tuesday</td>
<td>100</td>
</tr>
<tr>
<td>Wednesday</td>
<td>98</td>
</tr>
<tr>
<td>Thursday</td>
<td>96</td>
</tr>
<tr>
<td>Friday</td>
<td>94</td>
</tr>
<tr>
<td>Saturday</td>
<td>77</td>
</tr>
<tr>
<td>Sunday</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 4.5: Hourly peak load in percent of daily peak

<table>
<thead>
<tr>
<th>Hour of week 1, (%)</th>
<th>Weekdays</th>
<th>Hour of week 1, (%)</th>
<th>Weekdays</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 - 1 am</td>
<td>67</td>
<td>noon - 1 pm</td>
<td>95</td>
</tr>
<tr>
<td>1 - 2</td>
<td>63</td>
<td>1 - 2</td>
<td>95</td>
</tr>
<tr>
<td>2 - 3</td>
<td>60</td>
<td>2 - 3</td>
<td>93</td>
</tr>
<tr>
<td>3 - 4</td>
<td>59</td>
<td>3 - 4</td>
<td>94</td>
</tr>
<tr>
<td>4 - 5</td>
<td>59</td>
<td>4 - 5</td>
<td>99</td>
</tr>
<tr>
<td>5 - 6</td>
<td>60</td>
<td>5 - 6</td>
<td>100</td>
</tr>
<tr>
<td>6 - 7</td>
<td>74</td>
<td>6 - 7</td>
<td>100</td>
</tr>
<tr>
<td>7 - 8</td>
<td>86</td>
<td>7 - 8</td>
<td>96</td>
</tr>
<tr>
<td>8 - 9</td>
<td>95</td>
<td>8 - 9</td>
<td>91</td>
</tr>
<tr>
<td>9 - 10</td>
<td>96</td>
<td>9 - 10</td>
<td>83</td>
</tr>
<tr>
<td>10 - 11</td>
<td>96</td>
<td>10 - 11</td>
<td>73</td>
</tr>
<tr>
<td>11 - noon</td>
<td>95</td>
<td>11 - 12</td>
<td>63</td>
</tr>
</tbody>
</table>
4.2 Twenty-Four-Bus IEEE Reliability Test System

The twenty-four bus IEEE reliability test system is shown in figure 4.2 [2].

The used data can be found in [32] and [33]. Generating units data is presented in table 4.6 and data for transmission lines is shown in table 4.7. Bus 113 is assumed to be a slack bus. The estimated load is taken from [32] and is shown in tables 4.5, 4.4 and 4.8, it is assumed to be constant during the hour.

<table>
<thead>
<tr>
<th>ID</th>
<th>Unit Size, (MW)</th>
<th>Ramp Rate, (MW/Min)</th>
<th>Cost, ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>40</td>
<td>6</td>
<td>130</td>
</tr>
<tr>
<td>G2</td>
<td>152</td>
<td>4</td>
<td>16,1</td>
</tr>
<tr>
<td>G3</td>
<td>40</td>
<td>6</td>
<td>130</td>
</tr>
<tr>
<td>G4</td>
<td>152</td>
<td>4</td>
<td>16,1</td>
</tr>
<tr>
<td>G5</td>
<td>300</td>
<td>21</td>
<td>43,7</td>
</tr>
<tr>
<td>G6</td>
<td>591</td>
<td>9</td>
<td>48,6</td>
</tr>
<tr>
<td>G7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G8</td>
<td>60</td>
<td>5</td>
<td>56,6</td>
</tr>
<tr>
<td>G9</td>
<td>155</td>
<td>3</td>
<td>12,4</td>
</tr>
<tr>
<td>G10</td>
<td>155</td>
<td>3</td>
<td>12,4</td>
</tr>
<tr>
<td>G11</td>
<td>400</td>
<td>20</td>
<td>5,65</td>
</tr>
<tr>
<td>G12</td>
<td>400</td>
<td>20</td>
<td>5,65</td>
</tr>
<tr>
<td>G13</td>
<td>300</td>
<td>0</td>
<td>0,001</td>
</tr>
<tr>
<td>G14</td>
<td>310</td>
<td>6</td>
<td>12,4</td>
</tr>
<tr>
<td>G15</td>
<td>350</td>
<td>4</td>
<td>11,9</td>
</tr>
</tbody>
</table>
Figure 4.2: IEEE 24 bus reliability test system [2]
Table 4.7: Transmission lines data - 24 bus system

<table>
<thead>
<tr>
<th>ID</th>
<th>From Bus</th>
<th>To Bus</th>
<th>X, (pu)</th>
<th>Con, (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1011</td>
<td>1</td>
<td>2</td>
<td>0.014</td>
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<td>400</td>
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<td>500</td>
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<td>21</td>
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<td>500</td>
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<td>21</td>
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<td>22</td>
<td>0.068</td>
<td>500</td>
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Table 4.8: Load data - 24 bus system

<table>
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<tr>
<th>Bus</th>
<th>Peak load of one year, (MW)</th>
<th>Peak load of week 1, (%)</th>
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<td>108</td>
<td>86.2</td>
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<tr>
<td>2</td>
<td>97</td>
<td>86.2</td>
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<td>3</td>
<td>180</td>
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<td>74</td>
<td>86.2</td>
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<td>71</td>
<td>86.2</td>
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<tr>
<td>6</td>
<td>136</td>
<td>86.2</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
<td>86.2</td>
</tr>
<tr>
<td>8</td>
<td>171</td>
<td>86.2</td>
</tr>
<tr>
<td>9</td>
<td>175</td>
<td>86.2</td>
</tr>
<tr>
<td>10</td>
<td>195</td>
<td>86.2</td>
</tr>
<tr>
<td>13</td>
<td>265</td>
<td>86.2</td>
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<td>19</td>
<td>181</td>
<td>86.2</td>
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<td>20</td>
<td>128</td>
<td>86.2</td>
</tr>
</tbody>
</table>
Chapter 5

Case Studies. Results and Discussions

The six-bus test system was designed to make the detailed analysis of predictable results and therefore, to check the plausibility of the proposed idea.

An example of the twenty-four-bus test system was used to show the possibility of secure operation of the system and to make the comparison between proposed short-run economic dispatch and existing dispatch methods (five minutes dispatch). Also the flexibility of the system is evaluated by determining System Flexibility Index (SFI).

5.1 Six-Bus Test System

5.1.1 Probabilistic Security Analysis

The probabilistic security analysis was done for 6 bus system based on the previously performed data.

The optimization problem was run for two cases: system without security and system with security. By system with security we mean a system prepared for possible contingencies. The studied contingencies are 1) loss of \(100 \text{ MW}\) of load in bus 113,
Table 5.1: Generation schedule for initial steady state equilibrium

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>224</td>
<td>224</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 5.2: Nodal prices for initial steady state equilibrium

<table>
<thead>
<tr>
<th>Bus</th>
<th>Without Security $\lambda$, ($$/MW)</th>
<th>With Security $\lambda$, ($$/MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>113</td>
<td>48,600</td>
<td>48,600</td>
</tr>
<tr>
<td>119</td>
<td>48,600</td>
<td>48,600</td>
</tr>
<tr>
<td>120</td>
<td>48,600</td>
<td>48,600</td>
</tr>
<tr>
<td>121</td>
<td>48,600</td>
<td>48,600</td>
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<tr>
<td>122</td>
<td>48,600</td>
<td>48,600</td>
</tr>
<tr>
<td>123</td>
<td>48,600</td>
<td>48,600</td>
</tr>
</tbody>
</table>

2) loss of generating unit in bus 123 (50 MW), 3) loss of generating unit in bus 122 (100 MW) with probability of 50% each.

The optimal generation schedule for the system without and with security was found. Table 5.1 presents the optimal dispatch of generating units for cases without security and with security for initial steady state equilibrium. As seen from the results the initial dispatch is the same for system with and without security. Consequently, nodal prices for initial steady state equilibrium are the same in both cases (see table 5.2). Furthermore, prices remain the same during the transition period. This is explained by large enough capacities of the lines (none of the lines are congested) and not starting the more expensive generating unit (the most expensive unit of the system is already used).
5.1. SIX-BUS TEST SYSTEM

5.1.2 Real-Time Balancing Market

The simulations were done for three thousands minutes with one minute time interval.

Analysis of Optimal Dispatch for One Minute Dispatch Interval

Optimal dispatch for one minute dispatch interval of the studied system is shown in figure 5.1. It is easy to see that with the load increasing, the dispatch cost increases, and with reduction of the load, the dispatch cost decreases. Furthermore, the dispatch cost decreases while load is stable (transition period).

For better understanding, let’s look closer at the results of the simulation periods 1804 - 1814 (Fig. 5.2). At time period 1804, total load is 276.87 MW. To cover this load, the three cheapest generators, $G_2$, $G_3$ and $G_5$ are dispatched. Generation units $G_2$ and $G_3$ are used at their full capacity. The system price of this period is $11.9 / MWh. At the time period 1805, the load increases by 91.27 MW. To compensate this load change generating unit $G_5$ increases its generation due to its ramp rate by 4 MW, however it is not enough. Therefore generator $G_4$ starts producing 6 MW. Finally, expensive but fast (high ramp rate) generating unit $G_1$ covers the rest of the load. During the next periods 1807 - 1813, generators $G_4$ and $G_5$ continue to increase their levels of generation according to their ramp rates and generator $G_1$ decreases its generation. In time period 1813, generation units $G_2, G_3, G_4$ and $G_5$ are fully used and the dispatch cost achieves its possible minimum. Therefore the transition period is finished and the system is in the new steady state equilibrium (i.e. period 1814). The system price for transition period and the new steady state equilibrium is $48.6 / MWh.

Results of system price for the studied periods are shown in figure 5.3. During peak
Figure 5.1: Optimal dispatch for one minutes dispatch interval. Bottom figure is magnification of the top figure in time period range 2880 - 2945
hours (day time), price is always high as expensive generation unit $G_5$ is permanently used to keep balance in the system. During hours with low load system, the price fluctuates a lot as expensive generating unit $G_5$ is temporarily used only until cheaper generating units can cover the load (see bottom figure in figure 5.3).

**System Flexibility Analysis**

The six-bus system was also tested on flexibility. In the above example there are some periods where, due to too low ramp-rates, generation can not meet the demand as shown in figure 5.1 (see period 2942).

Two different systems are studied (figure 5.4):

- System A, that has the same generating units data and transmission lines data as in the above example (tables 4.1 and 4.2).
5.1. SIX-BUS TEST SYSTEM

Figure 5.3: System price. Bottom figure is magnification of the top figure in time period range 1750 - 1815

Table 5.3: Generating units data, System B

<table>
<thead>
<tr>
<th>ID</th>
<th>Unit Size, (MW)</th>
<th>Ramp Rate, (MW/min)</th>
<th>Cost, ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>291</td>
<td>50</td>
<td>48.6</td>
</tr>
<tr>
<td>G2</td>
<td>150</td>
<td>6</td>
<td>5.65</td>
</tr>
<tr>
<td>G3</td>
<td>100</td>
<td>6</td>
<td>0.001</td>
</tr>
<tr>
<td>G4</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>G5</td>
<td>50</td>
<td>4</td>
<td>11.9</td>
</tr>
</tbody>
</table>

- System B, that has reduced number of generators and reduced ramp rates of generating units, see table 5.3.

Both systems were tested for three levels of load changes: ±5%, ±10% and ±30% of the estimated load.

The simulations show predictable results. System A that contains more generating units with larger ramp rates has higher SFI than system B.

Results of both systems show that the system is not able to change its generation
5.2. TWENTY-FOUR-BUS IEEE TEST SYSTEM

Figure 5.4: Flexibility of the 6 bus system

level immediately under large contingency.

5.2 Twenty-Four-Bus IEEE Test System

5.2.1 Probabilistic Security Analysis

The load data is corresponding to a Tuesday of week 1 at 6 pm [32].

The modification from the data in [32] and presented above is the reduction of the capacities of lines \textit{1181}, \textit{1241} and \textit{1261} from 500 MW to 175 MW, 60 MW and 175 MW respectively.

The optimization problem was solved in order to find an optimal generation schedule
Table 5.4: Generation schedule for initial steady state equilibrium

<table>
<thead>
<tr>
<th>Generator</th>
<th>Without Security ( G, ) (MW)</th>
<th>With Security ( G, ) (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>25,245</td>
<td>40</td>
</tr>
<tr>
<td>G2</td>
<td>152</td>
<td>152</td>
</tr>
<tr>
<td>G3</td>
<td>0</td>
<td>19,466</td>
</tr>
<tr>
<td>G4</td>
<td>152</td>
<td>152</td>
</tr>
<tr>
<td>G5</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>G6</td>
<td>380,349</td>
<td>384,679</td>
</tr>
<tr>
<td>G8</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>G9</td>
<td>0</td>
<td>36,042</td>
</tr>
<tr>
<td>G10</td>
<td>150,561</td>
<td>54,922</td>
</tr>
<tr>
<td>G11</td>
<td>400</td>
<td>380</td>
</tr>
<tr>
<td>G12</td>
<td>329,845</td>
<td>365,889</td>
</tr>
<tr>
<td>G13</td>
<td>300</td>
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<td>G14</td>
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<td>310</td>
</tr>
<tr>
<td>G15</td>
<td>350</td>
<td>350</td>
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<tr>
<td>Dispatch</td>
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<td>58064,868</td>
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<tr>
<td>Cost, ($)</td>
<td>53776,154</td>
<td>58064,868</td>
</tr>
</tbody>
</table>

for the system under three possible contingencies. The studied contingencies are:

1. loss of the load in bus 16 (100 MW),
2. loss of generating unit in bus 16 (155 MW),
3. loss of generating unit in bus 15 (60 MW),

with probability of 50% each.

Table 5.4 presents the optimal dispatch of generating units and dispatch costs for cases without security and with security for initial steady state equilibrium. As seen from the results, to ensure security operation of the system under three possible contingencies generators 1, 3, 6, 8, 9 and 12 have to increase their generation levels compared to the case without security and generators 10 and 11 decrease it by the same amount. This means that the system has to run more expensive units in order to leave headroom in other generators for covering possible contingency.
### 5.2. TWENTY-FOUR-BUS IEEE TEST SYSTEM

Table 5.5: Generation schedule for final steady state equilibrium

<table>
<thead>
<tr>
<th>Generator</th>
<th>Contingency 1</th>
<th>Contingency 2</th>
<th>Contingency 3</th>
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<td>27,913</td>
</tr>
<tr>
<td>G2</td>
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<td>152</td>
</tr>
<tr>
<td>G3</td>
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<td>0,110</td>
<td>0</td>
</tr>
<tr>
<td>G4</td>
<td>152</td>
<td>152</td>
<td>152</td>
</tr>
<tr>
<td>G5</td>
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<td>300</td>
<td>300</td>
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<tr>
<td>G6</td>
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<td>117,922</td>
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<tr>
<td>G11</td>
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<td>400</td>
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<td>310</td>
<td>310</td>
<td>310</td>
</tr>
<tr>
<td>G15</td>
<td>350</td>
<td>350</td>
<td>350</td>
</tr>
</tbody>
</table>

| Dispatch Cost, ($) | 52531,098 | 55136,026 | 53901.731 |

Figure 5.5: Dispatch cost - 24 bus system
The proposed generation schedule and dispatch costs for three possible contingencies for final steady state equilibrium is shown in table 5.5. Figure 5.5 and table 5.6 perform the evolution of the dispatch cost during contingency (time intervals 2 - 21 in the table). As seen from these figure and table, dispatch cost is reducing rapidly during first time intervals (1-4) due to replacement of expensive generating units by cheap ones. The final dispatch costs are lower than initial ones on 9.5% in case of contingency 1; 5% in case of contingency 2; and 7.2% in case of contingency 3.

Table 5.7 summarizes the nodal prices for initial steady state equilibrium. Columns
Table 5.7: Nodal prices for initial steady state equilibrium

<table>
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<td>7,188</td>
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<td>164,218</td>
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<td>32,499</td>
<td>163,299</td>
<td>130,800</td>
</tr>
<tr>
<td>21</td>
<td>5,650</td>
<td>171,435</td>
<td>165,785</td>
</tr>
<tr>
<td>22</td>
<td>6,905</td>
<td>171,412</td>
<td>164,507</td>
</tr>
<tr>
<td>23</td>
<td>37,641</td>
<td>161,250</td>
<td>123,609</td>
</tr>
<tr>
<td>24</td>
<td>313,334</td>
<td>172,209</td>
<td>-141,125</td>
</tr>
</tbody>
</table>

1 and 2 show nodal prices of the system without and with security respectively. The difference between the prices is called price of security and it is shown in column 3 of table 5.7. In case with security, the nodal prices are increased in all buses except buses 3 and 24. These two buses are winning from the possible contingency. The explanation of the reduction in nodal prices in these buses can be that the secure operation of the system requires to not use line 1241 on full capacity (166,556 MW out of 175). Furthermore nodal prices in buses 14-23 greatly increase.
The nodal prices for final steady state are close to the nodal prices of system without security in initial steady state equilibrium as shown in table 5.8.

Figures 5.6, 5.7 and 5.8 present evolution of nodal prices of the system with security over all studied periods (22 time intervals).

For better illustration of the optimal results, the nodal prices of buses 103 and 115 for all periods and three possible contingencies are shown in table 5.9. As seen from this table and Tab. 5.7, after contingency occurs nodal prices are coming close
5.2. TWENTY-FOUR-BUS IEEE TEST SYSTEM

Figure 5.6: Nodal prices - 24 bus system under contingency 1

Figure 5.7: Nodal prices - 24 bus system under contingency 2
to their values in the case without contingency in initial steady state equilibrium.

By changing optimal initial steady state equilibrium in the system with security, the system is prepared for possible contingency and therefore there is no loss of the load. On the other hand, the system without security is often not able to keep balance between generation and consumption. Additional simulations were done for each contingency for the system without security. If contingency 1 occurs the system is not able to reduce generation fast enough. Therefore generation is higher than consumption for three periods. If contingency 2 happens, generation in the system does not fully cover demand for four periods. However the system can handle loss of generator of contingency 3 the same period it occurred. This is reflected in nodal prices. Figures 5.9 and 5.10 present nodal prices of buses 3 and 15 of system with and without security under three contingencies.
Table 5.9: Nodal prices ($/MW) for buses 103 and 115

<table>
<thead>
<tr>
<th>Time</th>
<th>Bus 103</th>
<th></th>
<th></th>
<th></th>
<th>Bus 115</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_1$</td>
<td>$c_2$</td>
<td>$c_3$</td>
<td>$c_1$</td>
<td>$c_2$</td>
<td>$c_3$</td>
</tr>
<tr>
<td>1</td>
<td>173,358</td>
<td></td>
<td></td>
<td></td>
<td>171,498</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>847,555</td>
<td>279,163</td>
<td>84,560</td>
<td>-799,093</td>
<td>263,883</td>
<td>3,536</td>
</tr>
<tr>
<td>3</td>
<td>85,629</td>
<td>48,600</td>
<td>136,618</td>
<td>2,484</td>
<td>48,600</td>
<td>2,141</td>
</tr>
<tr>
<td>4</td>
<td>195,555</td>
<td>65,907</td>
<td>208,602</td>
<td>-1,867</td>
<td>64,760</td>
<td>0,312</td>
</tr>
<tr>
<td>5</td>
<td>208,602</td>
<td>171,102</td>
<td>208,602</td>
<td>0,312</td>
<td>56,600</td>
<td>0,312</td>
</tr>
<tr>
<td>6</td>
<td>211,690</td>
<td>171,102</td>
<td>208,602</td>
<td>2,299</td>
<td>56,600</td>
<td>0,312</td>
</tr>
<tr>
<td>7</td>
<td>211,690</td>
<td>173,199</td>
<td>208,602</td>
<td>2,299</td>
<td>56,600</td>
<td>0,312</td>
</tr>
<tr>
<td>8</td>
<td>211,690</td>
<td>173,199</td>
<td>208,602</td>
<td>2,299</td>
<td>32,351</td>
<td>0,312</td>
</tr>
<tr>
<td>9</td>
<td>211,690</td>
<td>173,199</td>
<td>208,602</td>
<td>2,299</td>
<td>56,600</td>
<td>0,312</td>
</tr>
<tr>
<td>10</td>
<td>211,690</td>
<td>190,179</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>11</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>12</td>
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<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>13</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>14</td>
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<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>15</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>16</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>17</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>18</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>19</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>20</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>21</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
<tr>
<td>22</td>
<td>211,690</td>
<td>204,149</td>
<td>208,602</td>
<td>2,299</td>
<td>12,400</td>
<td>0,312</td>
</tr>
</tbody>
</table>

$c_1, c_2, c_3$ – contingency 1, contingency 2, contingency 3 respectively

As seen from figures 5.9 and 5.10, nodal prices are lower during transition period (time interval 2 - 21 on figures) and final steady state equilibrium (time interval 22) in the system with security, especially if the system is under contingency 1 and 2. This compensates the expenses in most of buses (for example bus 115, figure 5.10) of the initial steady state equilibrium (time interval 1). However, nodal prices are almost the same in the system with and without security if contingency 3 happens. Furthermore, the nodal price in initial steady state equilibrium (time interval 1 on
Figure 5.9: Nodal prices of bus 103 of system with and without security under three contingencies. Bottom figure is magnification of the top figure in nodal price range 0 - 900.
Figure 5.10: Nodal prices of bus 115 of system with and without security under three contingencies. Bottom figure is magnification of the top figure in nodal price range -100 - 300.
figures) is much higher in most of buses in the system with security (see figure 5.10).

5.2.2 Real-Time Balancing Market

The simulations were done for three thousand minutes with one minute time interval.

Analysis of Optimal Dispatch for One Minute and Five Minutes Dispatch Interval

The simulations were done for two cases: optimal dispatch with five minutes dispatch interval and optimal dispatch with one minute dispatch interval. Both simulations are based on the previously described model and are using the same data for fifty hours. The result of simulations, dispatch cost, is presented in figure 5.11. The load for all time intervals is shown in figure 5.12. The green line on this figure represents estimated load in the spot market. Estimated load is taken as first fifty hours hourly load of week 1 in [32], it is assumed to be constant during the hour. The blue line shows the real load at the corresponding time interval (one minute). And, finally, the red line shows the load used by the model with five minutes dispatch interval, that reflects the real load when the system is in a steady state. During the transition period the load is assumed to be constant in this case. Therefore, the blue and red lines are crossing when the system is in steady state in both models, see figure 5.12. However, it is not true for total dispatch cost (figure 5.11), as initial conditions for the optimizations were different. As seen from figure 5.11 the blue and red lines disagree due to considered load changing in optimization for one minute dispatch. For five minutes dispatch the corrective actions by frequency controls are needed. Since frequency controls do not give an optimal solution, the actual dispatch cost with five
minutes dispatch interval will be higher than dispatch cost with one minute dispatch interval.

Figures 5.13 and 5.14 show optimal dispatch of available generation for one minute dispatch interval and five minutes dispatch interval respectively. First, when load changes generating units increase/decrease levels of generation according to theirs ramp-rates and cost. Usually, generators with high ramp rates have higher price,
therefore next periods the generation in these units decrease and increase in the cheaper ones, consequently production cost decreases. These changes in generation happens until some of the system limits are not exceeded and production cost reaches its possible minimum.
Figure 5.13: Optimal dispatch for one minutes dispatch interval. Bottom figure is magnification of the top figure in time period range 2880 - 2945
Figure 5.14: Optimal dispatch for five minutes dispatch interval. Bottom figure is magnification of the top figure in time period range 2880 - 2945.
System Flexibility Analysis

The twenty-four-bus system was also tested on flexibility. The simulations were done for 300 time periods.

Two systems, system A and system B were tested on winter load, shown in figure 5.15. System A has the same data as in previous example. System B does not contain generator 6 and ramp rates of generators 5, 11 and 12 are reduced to 10 MW/min.

Both systems were tested for three levels of load changes: ±5%, ±10% and ±30% of the estimated load.

The simulations show predictable results. System A that contains more generating units with larger ramp rates has higher SFI than system B.

Results of both systems show that systems are not able to change generation levels.
immediately under large contingency.
Chapter 6

Conclusions and Future Work

6.1 Conclusion

Optimal dispatch of both energy and balancing services requires operation of power system respecting its physical limits over very short time frames. Nowadays, the dispatch of balancing services is imperfect as transmission lines limits and ramp rates are typically ignored. Furthermore, dispatching of balancing services is economically inefficient.

This thesis develops a short-run economic dispatch for handling both high and low frequency contingencies in the power system, respecting its physical limits. The model contains three states of power system: (1) initial steady state, (2) transient state, (3) final steady state. In the first state, preventive actions that system operator can do to cope with contingencies are modeled. The transient and final states model corrective actions.

Two applications of the proposed model are discussed and tested on two examples.
(six-bus power system and twenty-four-bus power system). First application is probabilistic security analysis. The model finds optimal dispatch for preventive and corrective actions of the power system. Also the price of security is found since the proposed model is a linear-programming problem and Lagrange multipliers are available. The second application is the real-time balancing market. The performed simulation illustrates the optimal response of the system to fluctuation of load and wind power over fifty hours, using dispatch intervals of one minute and five minutes. It is shown that the system operation cost can be reduced by decreasing the dispatch interval.

6.2 Future Work

The proposed model is working with the minimum time interval (dispatch interval) equal to one minute. This limitation comes from the ramp rates data given in [32] where the minimum time interval for the ramp rate is one minute. In the suggested model we assume that generators increase their generation with the constant speed - ramp-rates (MW/min). However, the behavior of the generators during this minute is not included in the model. By studying generators’ behavior the dispatch interval could be further reduced and therefore the proposed model would be improved.

Also the proposed model can be improved by including unit commitment (including start-up time, start-up cost, and minimum on and off times).

In the proposed model we assume that the system is lossless, which surely is not realistic. Transmission lines can be used in an unsatisfactory state for fifteen minutes. Including transmission lines losses and possibility of unsatisfactory state of the lines could change results of the optimization.
Modeling real data of the real system for the studied time period would make the analysis more valuable. Also it could make it possible to compare results of the simulation with the real outputs of the system and therefore could show the reduction in the dispatch cost and system price.
Bibliography


Appendix A

Power Transfer Distribution Factors (PTDF)

PTDF (Power Transfer Distribution Factors) shows the effect of nodal injection on the power flow on lines [34].

In this section, determination of PTDF for a simple triangular system (three buses and three lines) is presented.

The active power balance:

\[
\begin{bmatrix}
B_{12} + B_{13} & -B_{12} & -B_{13} \\
-B_{12} & B_{12} + B_{23} & -B_{23} \\
-B_{13} & -B_{23} & B_{23} + B_{13}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
= 
\begin{bmatrix}
inj_1 \\
inj_2 \\
inj_3
\end{bmatrix}
\]  
(A.1)

Where \( B_{ij} \) is the susceptance of the line connecting bus \( i \) and \( j \); \( \delta_i \) and \( inj_i \) is voltage angle and power injection in bus \( i \) respectively.

The line flow:

\[
\begin{bmatrix}
B_{12} & -B_{12} & 0 \\
B_{13} & 0 & -B_{13} \\
0 & B_{23} & -B_{23}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
= 
\begin{bmatrix}
flow_{12} \\
flow_{13} \\
flow_{23}
\end{bmatrix}
\]  
(A.2)
With $\text{flow}_{23}$ the power flow on line between buses $i$ and $j$. Assuming that bus 1 is a slack bus the equations A.1 and A.2 reduced to:

$$
\begin{bmatrix}
B_{12} + B_{23} & -B_{23} \\
-B_{23} & B_{23} + B_{13}
\end{bmatrix}
\cdot
\begin{bmatrix}
\delta_2 \\
\delta_3
\end{bmatrix}
=
\begin{bmatrix}
inj_2 \\
inj_3
\end{bmatrix}
$$

(A.3)

$$
\begin{bmatrix}
-B_{12} & 0 \\
0 & -B_{13} \\
B_{23} & -B_{23}
\end{bmatrix}
\cdot
\begin{bmatrix}
\delta_2 \\
\delta_3
\end{bmatrix}
=
\begin{bmatrix}
\text{flow}_{13} \\
\text{flow}_{23}
\end{bmatrix}
$$

(A.4)

Linear relation between the injections and the flows is obtained by combining equations A.3 and A.4.

$$
\begin{bmatrix}
-B_{12} & 0 \\
0 & -B_{13} \\
B_{23} & -B_{23}
\end{bmatrix}
\cdot
\begin{bmatrix}
B_{12} + B_{23} & -B_{23} \\
-B_{23} & B_{23} + B_{13}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
inj_2 \\
inj_3
\end{bmatrix}
=
\begin{bmatrix}
\text{flow}_{12} \\
\text{flow}_{13} \\
\text{flow}_{23}
\end{bmatrix}
$$

(A.5)

The product of first two matrices of equation A.5 is called as PTDF matrix:

$$
\begin{bmatrix}
H_{2,12} & H_{3,12} \\
H_{2,13} & H_{3,13} \\
H_{2,23} & H_{3,23}
\end{bmatrix}
\cdot
\begin{bmatrix}
inj_2 \\
inj_3
\end{bmatrix}
=
\begin{bmatrix}
\text{flow}_{12} \\
\text{flow}_{13} \\
\text{flow}_{23}
\end{bmatrix}
$$

(A.6)

The PTDF matrix can be defined from system parameters, dispatch of generation and load is not required [34].
Appendix B

Area Pricing

This section describes modeling of pricing in the case of transmission limits between two areas.

B.1 Congestion Management

Congestion of the line means there is not sufficient capacity of the transmission line according to the market desire. In a perfect power system there is no congestion, all lines of the system have enough capacity so that the power system is able to produce energy in the cheapest generating units. However, when congestion occurs more expensive ones will have to be used in areas where the lines capacities are not sufficient. Therefore, the prices in areas can be different and will be calculated on both sides of the transmission [28].

B.2 Implicit Auctioning

In implicit auctioning power energy and corresponding transmission between areas traded simultaneously and are coupled. When the capacity of the line is lower than
required transmission, an extra fee is added to every bid until enough bids are too high to be utilized. The price in one area will be higher than in another. The income of this procedure goes to the system operator [28].

**B.3 Explicit Auctioning**

In explicit auctioning power energy and corresponding transmission between areas are separated. The system operator has to choose which transmission capacity (can be full or part of it) will be set out for the auction. When a line is congested prices in the corresponding areas will be different. This method is used in systems with different market structure on different sides of the transmission line [28].

**B.4 Counter Trading**

System operator creates fictitious additional capacity for the transmission by buying power in deficit area and and sell it on the other side. Thereby the system operator makes one area and the price will be the same on both sides of congested line. The system operator will pay for the congestion by paying a higher price for purchased energy compared to the price of sold energy [28].