On Accuracy of Conic Optimal Power Flow

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MASTER THESIS

On Accuracy of Conic Optimal Power Flow

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Declaration of Authorship

I, Shan Huang, declare that this thesis titled, 'On Accuracy of Conic Optimal Power Flow' and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
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Date: 2015/6/19
Nowadays, with the increasing need for security and economic operation of power systems, the optimal power flow (OPF) has been an essential and predominant tool for economic and operation planning of power systems.

A developed conic formulation for OPF has been proposed [1]. This model is based on the line based flow equations and can be transformed into the form of second order cone programming (SOCP). The SOCP formulation of OPF problem can be solved using interior point methods (IPMs) [2].

In this thesis, a study on the performance of this developed conic formulation for OPF is carried out. Firstly, the accuracy of SOCP formulation is studied. A more accurate model is developed. The model is obtained by modifying the phase angle constraints of SOCP formulation. The modified model can be solved using sequential conic programming method. A comparison of results from these two models is made on different test systems. Secondly, the SOCP formulation is applied to both small and large test systems. The results of SOCP formulation is compared with the results from PSS/E OPF. The performance of SOCP formulation has shown the accurate and effectiveness for solving OPF problems.
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## Contents

Declaration of Authorship ........................................... i

Abstract ........................................................................... ii

Acknowledgements ................................................................ iii

Contents ............................................................................ iv

List of Figures ........................................................................ vi

List of Tables ......................................................................... vii

Abbreviations ....................................................................... viii

1 Introduction ...................................................................... 1
  1.1 Historical background .................................................. 1
  1.2 Formulation of OPF problem ........................................... 2
    1.2.1 AC power flow equations ............................................ 3
    1.2.2 DC power flow equations ............................................ 4
  1.3 OPF methods ............................................................... 5
  1.4 Aim of the project ....................................................... 5
  1.5 Outline of thesis ......................................................... 5

2 Second Order Cone Formulation for OPF ......................... 7
  2.1 Line-flow based equations ............................................. 8
  2.2 Second-order cone programming .................................... 10
  2.3 Optimal power flow .................................................... 11

3 Study on accuracy of SOCP OPF formulation .................... 14
  3.1 Modified phase angle difference constrains ..................... 14
  3.2 Optimal power flow and sequential conic programming .... 15
  3.3 Test results and comparison with SOCP OPF results ........ 16

4 Simulation results and comparison with PSS/E .................. 25
  4.1 PSS/E ........................................................................ 25
  4.2 Simulation results ...................................................... 27
5 Conclusion and further work

5.1 Conclusion ....................................... 30
5.2 Further work ..................................... 31

Bibliography ........................................... 32
List of Figures

2.1 Equivalent circuit of a branch ............................ 8

3.1 Flow chart of solving the modified model ....................... 17

3.2 Results of voltage magnitudes of IEEE 30-bus under normal load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP) ........................................ 18

3.3 Results of voltage magnitudes of IEEE 30-bus under high load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP) ...... 18

3.4 Results of voltage magnitudes of IEEE 118-bus under normal load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP) 19

3.5 Results of voltage magnitudes of IEEE 118-bus under high load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP) 19

3.6 Results of voltage magnitudes of IEEE 2383-bus under normal load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP) 23

4.1 PSS/E OPF data structure ......................................... 27

4.2 IEEE 30-bus power flow diagram ............................... 28
### List of Tables

3.1 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER (MAT) of IEEE 118-bus under normal load operation ............................................. 17
3.2 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER (MAT) of IEEE 14-bus under normal load operation ............................................. 20
3.3 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER of IEEE 14-bus under high load operation ....................................................... 21
3.4 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER of IEEE 30-bus under normal load operation A ............................................. 21
3.5 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER of IEEE 30-bus under normal load operation B ............................................. 22
3.6 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER of IEEE 30-bus under high load operation (A) ............................................. 22
3.7 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER of IEEE 30-bus under high load operation (B) ............................................. 22
3.8 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER (MAT) of IEEE 118-bus under high load operation ............................................. 23
3.9 OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER (MAT) of IEEE 2383-bus under normal operation ............................................. 24
4.1 OPF results from SOCP OPF (SOCP) and PSS/E OPF of IEEE 30-bus (A) ................................................................................................................................. 29
4.2 OPF results from SOCP OPF (SOCP) and PSS/E OPF of IEEE 30-bus (B) ................................................................................................................................. 29
4.3 OPF results from SOCP OPF (SOCP) and PSS/E OPF of IEEE 2383-bus 29
Abbreviations

A  Bus incidence matrix
AA Modified bus incidence matrix
C  Loop incidence matrix
PG  Vector of bus real power generation
QG  Vector of bus reactive power generation
Pf  Vector of real line flow
Pf  Vector of reactive line flow
Ploss  Vector of real losses of branches
Qloss  Vector of reactive losses of branches
V2  Vector of square of bus voltage
B  Diagonal matrix with the diagonal elements are shunt of buses
R  Diagonal matrix with the diagonal elements are resistance of branches
X  Diagonal matrix with the diagonal elements are reactance of branches
δ  Phase angle of buses
Ynk  Bus admittance matrix whose nkth element
Vin  Imaginary part of bus voltage
Vk  Real part of bus voltage
Gnk  Real part of Ynk
Bnk  Imaginary part of Ynk
Chapter 1

Introduction

1.1 Historical background

Worldwide, to achieve the security and economic operation of power system is of great significance to the development of national economy. Over the past decades, with the growing of consumer demand, development of renewable power plants and some other factors, the need for finding efficient and reliable optimization models which can solve both security and economic issues of system operation is increasing [3]. In the 1960s, the node voltage constraints, branch flow constraints and other operation constraints were first introduced into the economic dispatch problem by Carpentier [4]. Although the problem not successfully solved, this nonlinear programming formulation introduced by Capentier was the first mathematical model of optimal power flow (OPF) problem.

Optimal power flow (OPF) seeks to find an optimal operational point of power system to satisfy an economic, planning and reliable objective subject to the power flow constraints and system control limits. OPF is developed as a predominant tool for economic and operation planning of power systems. OPF problem often is a kind of optimization problem with the character of non-linear, non-convex and larger scale [3]. Different formulations have been developed to obtain various objectives by using different control variables and constraints. However the optimizations of power flow problems always include the power flow equations as the constraints in the network. Moreover, different
types of optimization problems always have different names depending on the optimization programming format of formulations. The formulations can be classified as linear programming, nonlinear programming, quadratic programming and etc.

The solution methods for OPF problems also have been developed for decades. Many mathematical programming methods can be utilized to OPF problems, and some software are developed to solve OPF problems. But, due to the increasing size and complexity of power system, it is difficult to find a proper formulation and solution method for all OPF problems.

### 1.2 Formulation of OPF problem

The standard form of formulation of OPF problem can be represented as follows [5]:

\[
\begin{align*}
\min f(u, x) \\
\text{s.t.} g(u, x) &= 0 \\
h(u, x) &\leq 0
\end{align*}
\] (1.1)

Where \( f(u, x) \) represents the objective function which is the aim of optimization. Vector function \( g(u, x) \) and \( h(u, x) \) represent the equalities and inequalities of constraints. The vector \( u \) and \( x \) represent the state variables and controllable variable separately. The state variables can describe the state of power system and controllable variables representing the control equipment of system. Different state and controllable variables can be chosen according to the form of power flow equations and particular operation situation of the system.

The equalities of constrains \( g(u, x) \) are composed of power flow equations and other operation balance constrains. The form of power flow equations are various. The power flow equations is classified into two kinds: alternating current (AC) power flow equations and direct current (DC) power flow equations. The inequalities of constrains \( h(u, x) \) often contain upper and lower bounds of variables, such as voltage magnitude and phase angles.
1.2.1 AC power flow equations

AC power flow equations model the power system completely. Many kinds of AC power flow equations are present in literature. The normal form of AC power flow equations can be written as:

\[ P_n = Re(V_n \sum_{k=1}^{N} Y_{nk}^* \cdot V_{nk}^*) \]  \hspace{1cm} (1.2)

\[ Q_n = Im(V_n \sum_{k=1}^{N} Y_{nk}^* \cdot V_{nk}^*) \]  \hspace{1cm} (1.3)

The polar form of AC power flow equations are often used in OPF formulation. The variables are set as the voltage magnitudes and voltage phase angle.

\[ P_n = \sum_{k=1}^{N} |V_n||V_k||Y_{nk}|\cos(\delta_n - \delta_k - \gamma_{nk}) \]  \hspace{1cm} (1.4)

\[ Q_n = \sum_{k=1}^{N} |V_n||V_k||Y_{nk}|\sin(\delta_n - \delta_k - \gamma_{nk}) \]  \hspace{1cm} (1.5)

Another form called rectangular form of power flow equations are proposed in recent papers [6]:

\[ P_n = \sum_{k=1}^{N} G_{nk}(V_n^rV_k^r + V_n^iV_k^i) + B_{nk}(V_n^rV_k^i - V_n^iV_k^r) \]  \hspace{1cm} (1.6)

\[ Q_n = \sum_{k=1}^{N} G_{nk}(V_n^rV_k^r - V_n^iV_k^i) - B_{nk}(V_n^rV_k^i + V_n^iV_k^r) \]  \hspace{1cm} (1.7)

As can be seen from the equations, the variables are set as the real and imaginary part of voltages.

There are some other power flow equations whose variables are branch flow variables. [7] introduced a new set of power flow equations named branch flow model, which concerns the balances of branch flow. The variables of this model are current magnitude and angle, voltage magnitude and angle and real and reactive line flow of the system.
The feasible region of the OPF formulations including the AC power flow equations are nonlinear and non-convex. Due to such reason, it is difficult to find a global solution. Many researches into convexification of OPF formulations has been active for a long time. Paper [8] shows that the AC power flow equations can be relaxed using semidefinite programming (SDP). Some limitations of SDP relaxation for OPF problems are also analyzed for meshed networks in [8]. [9] proposed an approach for relaxing the branch flow equations for radial networks, i.e., relaxing the equalities to inequalities and obtaining a second order cone programming. In [10], two relaxations are used for solving the OPF problems for both meshed and radial networks. The voltage and current angle constrains are eliminated and the conic relaxation is used. These two relaxations are proven exact for radial networks, but for meshed networks, in order to obtain the global solution, some phase shifters should be used in the network.

1.2.2 DC power flow equations

The DC power flow equations also represent the AC power system, but it is simplified by using a linear form. To derive the DC power equations, some assumptions are made:

- The differences of voltage angles are small. \( \sin(\delta_n - \delta_k) = \delta_n - \delta_k, \cos(\delta_n - \delta_k) = 1 \).
- All the voltage magnitude are set to 1 p.u.
- Only line reactance are taken into consideration.

The DC power flow equations are written as follows:

\[
P_n = \sum_{k=1}^{N} B_{nk}(\delta_n - \delta_k)
\]  

(1.8)

Many OPF formulations use DC power flow equations due to the linear constraints which can be solved with fast speed and reliability. However, the DC power flow equations neglect the losses of lines and the solution may result in large errors when applied to large system [11].


1.3 OPF methods

The most popular methods for solving OPF problems are Newton method [12], sequential linear programming (SLP) [13], sequential quadratic programming (SQP) [14] and interior point methods (IPMs) [15]. Newton method, SLP, SQP are active set methods for solving OPF problems. The active set methods suffering the difficulty of identification of the active set of inequality constrains may have low efficiency when solving the OPF problem. IPMs do not have such question. IPMs reach the optimal solution by following a declined path of feasible region. IPMs is widely used and can be seen as the most efficient and fastest algorithm for solving OPF problems. Some literature have studied the enhancements over IPMs, Primal-Dual Interior Point Methods (PDIPMs) [16], Mehrotra’s predictor-corrector techniques [17], and trust region techniques [18].

1.4 Aim of the project

A conic formulation for OPF problems are developed in [19], [1]. The formulation is based on line flow equations and formed in a second order programming. We make full study on the SOCP formulation for OPF. To achieve this aim, the accuracy and efficiency are analyzed through applying this model to several systems, including large systems. Also, a comparison is made between results from SOCP formulation implemented in GAMS and different software.

1.5 Outline of thesis

In chapter 1, the historical background of OPF is introduced. The research on OPF formulation and solution methods are also described.

In chapter 2, the conic formulation of OPF based on line flow equations is illustrated. The non-linear equalities of line flow based equations can be replaced by inequalities and the formulation for OPF can be formed in SOCP.
In chapter 3, a study on accuracy of the conic formulation for OPF is carried out. A modified model which change the phase angle constrains of conic formulation is described. The modified model can obtain a more accurate solution compared with conic formulation. The comparison is analyzed between these two models.

In chapter 4, the performances of SOCP formulation is studied on both small and large test systems. The comparison of the results from SOCP formulation and the results from PSS/E OPF is studied.
Chapter 2

Second Order Cone Formulation for OPF

As the development of the optimal power flow formulation, a second order cone programming formulation for OPF in a meshed network has been developed [19]. The developed conic programming formulation is based on line flow based equations which has been first described in [20]. Compared to conventional power flow models, the line flow based model can directly reflect the line power flows of power system. The line flow based equations have been studied in many papers. Since the nonlinearity of the equations of branch losses, [21] has neglected the equations representing the line losses. [22] used the approximated methods to linear the line losses. But in [19], all the nonlinear equations of real and reactive losses are taken into consideration.

In order to develop the nonconvex OPF problem to convex optimization problem, some approximations are used to linearize the angle phase constrains. Besides, the nonlinear equations of power losses are replaced by relaxing the equalities into inequalities which can be formed as rotated quadratic cones. By using these methods, the problem which is formulated by using the line flow variables is formed as a second order programming problem. The problem can be efficiently solved through polynomial time interior point methods [2].

Also, the author has proven the accuracy of the proposed OPF problem by comparing the results of SOCP formulation with the results obtained from MATPOWER. The tightness of the convex relaxation was also examined and proved.
In this section, the SOCP formulation based on line flow based equations will be described in details.

### 2.1 Line-flow based equations

First, the interrelation of voltages and phase angles of a branch can be derived. An equivalent circuit of a branch of system is shown in figure 1 and we define the line flow direction is from bus $i$ to bus $j$. The voltage drop from each branch can be written as

$$V_i \angle \delta_i = V_j \angle \delta_j + P_f - jQ_f (R_f + jX_f) \tag{2.1}$$

Then calculate the square of the magnitude of both sides and the following equation can be obtained

$$V_i^2 V_j^2 = V_j^4 + 2V_j^2 (R_f P_f + X_f Q_f) + (P_f^2 + Q_f^2) (R_f^2 + X_f^2) \tag{2.2}$$

Diving \(2.2\) by $V_j^2$, \(2.3\) is obtained as

$$V_i^2 - V_j^2 = 2 (RP_f + XQ_f) + P_{loss} R_f + Q_{loss} X_f \tag{2.3}$$

Where

$$P_{loss,f} = \frac{P_f^2 + Q_f^2}{V_j^2} R_f \tag{2.4}$$

$$Q_{loss,f} = \frac{P_f^2 + Q_f^2}{V_j^2} X_f \tag{2.5}$$

The interrelation of phase angle across a branch can be written as

$$V_i V_j \sin (\delta_i - \delta_j) = X_f P_f - R_f Q_f \tag{2.6}$$
Consider a connected graph $W$ which can represent a meshed power system with $n$ nodes which represent the buses and $l$ links which represent the lines in the network. We regard the graph $W$ as a directed graph and define the orientation as the smaller number node points to the bigger number node of a link, i.e. a link $f$ is connected by $(i, j)$, if $i < j$, the direction of link is from node $i$ to node $j$. So the direction of line flow is the same with the directed link. Choose a spinning tree $T$ of graph, the spinning tree includes $(n-1)$ links and the number of fundamental cycles of graph is $(l-n+1)$. According to graph theory, the line-flow based equations can be derived.

The active and reactive power balance at each bus can be written as follows: for all $i \in n$, $f \in l$,

$$P_{Gi} - P_{Li} = \sum_{f=1}^{l} A(i, f) \cdot P_f + \sum_{f=1}^{l} A'(i, f) \cdot P_{loss,f} \tag{2.7}$$

$$Q_{Gi} - Q_{Li} = \sum_{f=1}^{l} A(i, f) \cdot Q_f + \sum_{f=1}^{l} A'(i, f) \cdot Q_{loss,f} - B(i, i) V_i^2 \tag{2.8}$$

The elements of matrix $A$ and $AA$ are defined as

$$A_{il} = \begin{cases} 1, & \text{if line } l \text{ leaves bus } i \\ -1, & \text{if line goes into bus } i \\ 0, & \text{otherwise} \end{cases} \tag{2.9}$$

$$A'_{il} = \begin{cases} 1, & \text{if line } l \text{ leaves bus } i \\ 0, & \text{otherwise} \end{cases} \tag{2.10}$$

Since 2.6 is a trigonometric function, some assumptions are made to linearized it. Assuming $\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j$ and $V_i V_j \approx 1$, 2.6 can be written as

$$\delta_i - \delta_j = X(f, f) P_f - R(f, f) Q_f \tag{2.11}$$

In a meshed network, the sum of phase angle drops around fundamental cycles is zero. So the equations of cycle phase angle can be obtained by neglecting the approximations used in 2.11.

$$0 = CXP - CRQ \tag{2.12}$$

A fundamental cycle is consisted of a line that does not belong to the spinning tree together with the path in $T$. We define the direction of each fundamental cycle is the
same with the direction of the line outside $T$. So $C$ can be partitioned into:

$$C = [C_T \ C_\bot] \quad (2.13)$$

where $C_T$ is $(n - 1) \times (l - n + 1)$ corresponding to the lines included in $T$ and $C_\bot$ is $(l - n + 1) \times (l - n + 1)$ corresponding to the lines which do not belong to $T$. $C_\bot$ is a diagonal matrix with all the diagonal elements equal to 1. $C_\bot$ can be calculated by

$$C_T = (-A_T^{-1}A_\bot^*)_T \quad (2.14)$$

$A^*$ is a reduced incidence matrix which is derived by removing one row of $A$.

Therefore, a meshed network is described by 2.3, 2.4, 2.5, 2.7, 2.8 and 2.12.

### 2.2 Second-order cone programming

The second-order cone programming problem is a kind of convex programming problems, which minimize a linear function subject to a set of linear constraints and nonlinear constraints which are in form of convex cones. The SOCP problems can be solved efficiently by using polynomial time interior point methods. The standard form of SOCP is as follows [2]:

$$\begin{align*}
\text{Min} & \quad c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad x_{\min} \leq x \leq x_{\max} \\
& \quad x \in C
\end{align*} \quad (2.15)$$

where $x \in \mathbb{R}^n$, $C$ is to be a convex cone, $c$ and $b$ are the constant number.

Put the variables $x$ into different sets $S^p$, $p = 1, \ldots, k$, and each variable $x$ is only contained in one set $S^p$. Define

$$C = \{x \in \mathbb{R}^n : x_{sp} \in C_p, p = 1, \ldots, k\} \quad (2.16)$$

where $x_{sp}$ is defined as the variables which are include in set $S^p$. And $C_p$ must be fulfilled the one of these two following forms:
Chapter 2. Second Order Cone Formulation for OPF

Quadratic cone:

\[ C_p = \{ x \in \mathbb{R}^{n_p}, : x_1 \geq \sqrt{\sum_{i=2}^{n_p} x_i^2} \} \] (2.17)

Rotated quadratic cone:

\[ C_p = \{ x \in \mathbb{R}^{n_p}, : 2x_1x_2 \geq \sum_{i=3}^{n_p} x_i^2, x_1, x_2 \geq 0 \} \] (2.18)

If the problem is constrained by linear equalities and quadratic inequalities which can be formed as these two types of cones, the problem can be formed in SOCP and solved by using interior point methods [2].

2.3 Optimal power flow

The line flow based equations are used to formulate the optimal power flow. The objective function is set to minimize the fuel cost of all the generators in the system. Since the objective function should be formed in a linear form, the piecewise linear cost model of generators is considered. The objective function is written as

\[ \text{Min} \quad c_0 P_{Gi} + c_1 \] (2.19)

where, the constant \( c_0, \ c_1 \) are the coefficients with the piecewise linear cost model of generators. The vector of variables of the proposed line flow based model is defined as

\[
X = \begin{bmatrix}
V^{2^T} \\
P^T \\
Q^T \\
P_{Gi}^T \\
Q_{Gi}^T \\
P_{loss}^T \\
Q_{loss}^T
\end{bmatrix}
\] (2.20)
Chapter 2. Second Order Cone Formulation for OPF

But the feasible set of the OPF problem is nonconvex due to the quadratic equation of 2.4 and 2.5. And can be deduced from by using the following equation.

\[ Q_{\text{loss},f} = \frac{X(f,f)}{R(f,f)} P_{\text{loss},f} \]  

(2.21)

Because \( V_j^2 \geq 0 \), \( P_{\text{loss},f} \geq 0 \), relax 2.4 into inequalities

\[ P_{\text{loss},f} \geq \frac{P_j^2 + Q_j^2}{V_j^2} R(f,f) \]  

(2.22)

Form 2.4 as a rotated quadratic cone

\[ 2V_j^2 P_{\text{loss},f}^* \geq P_j^2 + Q_j^2 \]  

(2.23)

\[ P_{\text{loss},f}^* = \frac{P_{\text{loss},f}}{2R(f,f)} \]  

(2.24)

By transforming 2.4 to 2.23, the feasible set of this problem is convex.

Therefore, the equality and inequality constrains of the problem can be written as

\[ P_G - P_L - A \cdot P - AA \cdot P_{\text{loss}} = 0 \]  

(2.25)

\[ Q_G - Q_L - A \cdot Q - AA \cdot Q_{\text{loss}} - B \cdot Y = 0 \]  

(2.26)

\[ CXP - CRQ = 0 \]  

(2.27)

\[ A^T \cdot Y - 2RP + 2XQ + RP_{\text{loss}} + XQ_{\text{loss}} = 0 \]  

(2.28)

\[ Q_{\text{loss}} \cdot R - P_{\text{loss}} \cdot X = 0 \]  

(2.29)

\[ 2P_{\text{loss}}^* \cdot R = P_{\text{loss}} \]  

(2.30)

\[ 2V_j^2 P_{\text{loss},f}^* \geq P_j^2 + Q_j^2 \]  

(2.31)

\[ V_j^2 \leq V_j^2 \leq V_{j}^2 \]  

(2.32)

\[ P_{Gi} \leq P_{Gi} \leq P_{Gi} \]  

(2.33)

\[ Q_{Gi} \leq Q_{Gi} \leq Q_{Gi} \]  

(2.34)

\[ P_i \leq P_i \leq P_i \]  

(2.35)
where the underline and overline represent the upper and lower bound of these line flow variables.
Chapter 3

Study on accuracy of SOCP OPF formulation

In the proposed SOCP OPF formulation, in order to linearize the phase angle drops across a line, some approximations are used. However, this will affect the accuracy of the solution results. Can we find a proper way to linearize the phase angle constrains more accurately. To increase the accuracy of the OPF results, a modified model is illustrated in this section. The model is obtained by modifying the phase angle constraints in the SOCP OPF formulation. In this modified model, the interrelations of phase angles are described as functions of line flow variables and linearize the functions by using Taylor’s series. The modified model can be solved using sequential conic programming method as used in [23]. This means that an external loop is added to the previous model. Although some approximations are still used because of truncating the high order terms of Taylor’s series, the solution will be more accurate and closer to the exact operation point through the iterative loops.

3.1 Modified phase angle difference constrains

We can obtain the imaginary part and real part of both sides in 2.1 as follows

\[ V_i V_j \sin(\delta_i - \delta_j) = X_l P_f - R_l Q_f \]  

(3.1)
\[ V_i V_j \cos(\delta_i - \delta_j) = R_i P_f + X_i Q_f + V_j^2 \] (3.2)

Then the phase angle difference through a branch can be obtained

\[ \delta_i - \delta_j = \tan^{-1} \left( \frac{X_i P_f - R_i Q_f}{R_i P_f + X_i Q_f + V_j^2} \right) \] (3.3)

The trigonometric functions are not compatible in the conic quadratic format, Equation 3.3 should be linearized. Define

\[ T_{ij} = X_i P_f - R_i Q_f \] (3.4)

\[ S_{ij} = R_i P_f + X_i Q_f + V_j^2 \] (3.5)

By following Taylor’s series and neglect the higher terms, Equation 3.3 can be expressed at an estimate point \((S_{in}^{(q)}, T_{in}^{(q)})\).

\[ \delta_i - \delta_j + \frac{T_{ij}^{(q)}}{S_{ij}^{(q)^2} + T_{ij}^{(q)^2}} S_{ij} - \frac{S_{ij}^{(q)}}{S_{ij}^{(q)^2} + T_{ij}^{(q)^2}} T_{ij} = \tan^{-1} \frac{S_{ij}^{(q)}}{T_{ij}^{(q)}} \] (3.6)

Equation 3.6 is a linearization of Equation 3.3, which shows the interrelations between phase angles in a more accurate way instead of using the approximations in 2.11.

### 3.2 Optimal power flow and sequential conic programming

The objective function is the same with the one of SOCP OPF formulation. The variables of modified model are defined as
Because of including the linearized angle constraints (50), an iterative procedure of solving the problem is necessary. At each iteration, the formulation can be formed as a SOCP programming problem. The conic programming OPF formulation is formed as follows.

Minimize $2.19$

Subject to: $2.25, 2.26, 2.28, 2.29, 2.30, 2.31, 2.32, 2.33, 2.34, 2.35, 2.36, 3.4, 3.5, 3.6$.

The process can be done by: first set $q$ equal to 0 and give initial values to $S_{ij}^0$ and $T_{ij}^0$. Second, solve the conic formulation defined above, increase $q$ by 1, the results can be obtained and denote the results $S_{ij}^q$, $T_{ij}^q$. If $\|S_{ij}^q - S_{ij}^{(q-1)}\| < \epsilon$ and $\|T_{ij}^q - T_{ij}^{(q-1)}\| < \epsilon$, the final results of problem are derived, otherwise, repeat the step two.

Figure 3.1 shows the flow chart of the process solving the modified model.

### 3.3 Test results and comparison with SOCP OPF results

The accuracy comparison between the results of modified model and SOCP model is carried out by comparing with the results of the standard OPF formulation. The the modified model and SOCP model is programmed in the platform GAMS and the MOSEK [24] solver is used. The standard OPF formulation is carried out in MATPOWE. The different OPF models are applied to three test systems IEEE14-bus, IEEE 30-bus, IEEE 118-bus and IEEE 2383-bus. In order to investigate the differences of results from these
two models, we modify the small test systems by increasing the load to maximum value which a feasible solution can be obtained (since the load has a great impact on the voltage and phase angle).

| Table 3.1: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER (MAT) of IEEE 118-bus under normal load operation |
|-------------------------------|-----------------|-----------------|-----------------|
|                              | MAT             | SOCP            | MM              |
| Total cost [§]               | 86331.14        | 86334.08        | 86331.14        |
| Total P losses [MW]          | 74.557          | 74.704          | 74.557          |
| Total Q losses [MVAR]        | 438.62          | 439.555         | 438.634         |
| Total P gen [MW]             | 4316.56         | 4316.704        | 4316.557        |
| Total Q gen [MVAR]           | 301.91          | 302.625         | 301.917         |
| Elapsed Time [s]             | 0.89            | 0.543           | 2.729           |

For IEEE 14-bus, the results from three models are shown in table 3.2 and 3.3. The
Chapter 3. *Study on accuracy of SOCP OPF formulation*

Figure 3.2: Results of voltage magnitudes of IEEE 30-bus under normal load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP)

Figure 3.3: Results of voltage magnitudes of IEEE 30-bus under high load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP)
Figure 3.4: Results of voltage magnitudes of IEEE 118-bus under normal load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP)

Figure 3.5: Results of voltage magnitudes of IEEE 118-bus under high load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP)
Table 3.2: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER (MAT) of IEEE 14-bus under normal load operation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAT</td>
<td>SOCP</td>
<td>MM</td>
</tr>
<tr>
<td>1</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
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<tr>
<td>2</td>
<td>1.052</td>
<td>1.052</td>
<td>1.052</td>
</tr>
<tr>
<td>3</td>
<td>1.019</td>
<td>1.019</td>
<td>1.019</td>
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<tr>
<td>4</td>
<td>1.017</td>
<td>1.017</td>
<td>1.017</td>
</tr>
<tr>
<td>5</td>
<td>1.022</td>
<td>1.022</td>
<td>1.022</td>
</tr>
<tr>
<td>6</td>
<td>1.018</td>
<td>1.018</td>
<td>1.018</td>
</tr>
<tr>
<td>7</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
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<tr>
<td>8</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td>9</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>10</td>
<td>0.994</td>
<td>0.994</td>
<td>0.994</td>
</tr>
<tr>
<td>11</td>
<td>1.002</td>
<td>1.002</td>
<td>1.002</td>
</tr>
<tr>
<td>12</td>
<td>1.002</td>
<td>1.002</td>
<td>1.002</td>
</tr>
<tr>
<td>13</td>
<td>0.997</td>
<td>0.997</td>
<td>0.997</td>
</tr>
<tr>
<td>14</td>
<td>0.978</td>
<td>0.978</td>
<td>0.978</td>
</tr>
</tbody>
</table>

Elapsed time [s]  MAT | SOCP | MM
0.61 | 0.512 | 1.394

results which are from modified model is closer to the standard results than those which are from the SOCP formulation. But the differences from modified model and SOCP formulation are quite small. For IEEE 30-bus system under both normal load and high operation, from table 3.4, 3.5, 3.6 and 3.7, it can be seen that the results from SOCP and modified model are still close to each other, compared to the standard results from MATPOWER, the results from modified model are more accurate. Furthermore, the modified model has reduced the voltage magnitude errors, figure 3.2 shows that the voltage magnitudes of bus node number 25 and 29 are corrected by the modified model when network is under normal load operation. Similarly, when the network under high load operation, figure 3.3 shows that the voltage magnitudes of bus node number 10 and 22 are corrected.

The results from different models of IEEE 118-bus are shown in table 3.1 and 3.8. The modified model still gets better results. And the accuracy of voltage magnitudes is increased by the modified model. Since it is hard to display all the bus nodes in one figure, only 55 bus nodes are chosen to show in figure 3.4 and 3.5. The maximum voltage
### Table 3.3: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER of IEEE 14-bus under high load operation

<table>
<thead>
<tr>
<th>Bus</th>
<th>Voltage [p.u]</th>
<th>( P_G ) [MW]</th>
<th>( Q_G ) [MVAR]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAT SOCP MM</td>
<td>MAT SOCP MM</td>
<td>MAT SOCP MM</td>
</tr>
<tr>
<td>1</td>
<td>1.06 1.06 1.06</td>
<td>54.02 54.02 54.02</td>
<td>0 0 0</td>
</tr>
<tr>
<td>2</td>
<td>1.06 1.06 1.06</td>
<td>129.65 129.65 129.65</td>
<td>46.17 46.17 46.17</td>
</tr>
<tr>
<td>3</td>
<td>1.039 1.039 1.039</td>
<td>92.85 92.85 92.85</td>
<td>40 40 40</td>
</tr>
<tr>
<td>4</td>
<td>1.024 1.024 1.024</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>5</td>
<td>1.031 1.031 1.031</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>6</td>
<td>1.009 1.009 1.009</td>
<td>81.49 81.49 81.49</td>
<td>24 24 24</td>
</tr>
<tr>
<td>7</td>
<td>1.002 1.002 1.002</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>8</td>
<td>1.041 1.041 1.041</td>
<td>36.15 36.15 36.15</td>
<td>24 24 24</td>
</tr>
<tr>
<td>9</td>
<td>0.968 0.968 0.968</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>10</td>
<td>0.962 0.962 0.962</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>11</td>
<td>0.979 0.979 0.979</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>12</td>
<td>0.984 0.984 0.984</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>13</td>
<td>0.974 0.974 0.974</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>14</td>
<td>0.94 0.94 0.94</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elapsed time [s]</th>
<th>MAT SOCP MM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.62 0.538 1.853</td>
</tr>
</tbody>
</table>

For the larger test case IEEE 2383-bus system, from table 3.9, the same with the other three cases, the accuracy of results is increased by the modified model. In this case, the
Table 3.5: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER of IEEE 30-bus under normal load operation B

<table>
<thead>
<tr>
<th></th>
<th>MAT</th>
<th>SOCP</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost [$]</td>
<td>316.38</td>
<td>316.39</td>
<td>316.38</td>
</tr>
<tr>
<td>Total P losses [MW]</td>
<td>3.992</td>
<td>3.988</td>
<td>3.992</td>
</tr>
<tr>
<td>Total Q losses [MVAR]</td>
<td>15.06</td>
<td>15.051</td>
<td>15.058</td>
</tr>
<tr>
<td>Total P gen [MW]</td>
<td>193.19</td>
<td>193.188</td>
<td>193.192</td>
</tr>
<tr>
<td>Total Q gen [MVAR]</td>
<td>80.07</td>
<td>80.059</td>
<td>80.067</td>
</tr>
<tr>
<td>Elapsed Time [s]</td>
<td>0.72</td>
<td>0.526</td>
<td>2.065</td>
</tr>
</tbody>
</table>

Table 3.6: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER of IEEE 30-bus under high load operation (A)

<table>
<thead>
<tr>
<th>Gen</th>
<th>$P_G$ [MW]</th>
<th>$Q_G$ [MVAR]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAT</td>
<td>SOCP</td>
<td>M</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>22</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>23</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>27</td>
<td>38.557</td>
<td>38.56</td>
</tr>
</tbody>
</table>

Table 3.7: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER of IEEE 30-bus under high load operation (B)

<table>
<thead>
<tr>
<th></th>
<th>MAT</th>
<th>SOCP</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost [$]</td>
<td>685.31</td>
<td>685.29</td>
<td>685.31</td>
</tr>
<tr>
<td>Total P gen [MW]</td>
<td>318.56</td>
<td>318.56</td>
<td>318.55</td>
</tr>
<tr>
<td>Total Q gen [MVAR]</td>
<td>157.63</td>
<td>157.632</td>
<td>157.616</td>
</tr>
<tr>
<td>Elapsed Time [s]</td>
<td>0.74</td>
<td>0.542</td>
<td>2.153</td>
</tr>
</tbody>
</table>

The accuracy of voltage magnitudes has gotten greater increase as shown in figure 3.6 (The figure only shows a small part of buses.). The results of voltage magnitudes for modified model is much closer to the standard results. The maximum increase of accuracy is about 1.7%.

Also, it should be noted that the elapsed time of modified model is almost three times of that of SOCP formulation, which is because the external iterative procedure of modified
Figure 3.6: Results of voltage magnitudes of IEEE 2383-bus under normal load operation from MATPOWER (STRD), SOCP (SOCP) and modified model (SCP)

Table 3.8: OPF results from modified model (MM), SOCP OPF (SOCP) and MATPOWER (MAT) of IEEE 118-bus under high load operation

<table>
<thead>
<tr>
<th></th>
<th>MAT</th>
<th>SOCP</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost [$]</td>
<td>200160</td>
<td>200146</td>
<td>200150</td>
</tr>
<tr>
<td>Total P losses [MW]</td>
<td>176.943</td>
<td>177.323</td>
<td>176.948</td>
</tr>
<tr>
<td>Total Q losses [MVAR]</td>
<td>1020.02</td>
<td>1019.611</td>
<td>1019.255</td>
</tr>
<tr>
<td>Total P gen [MW]</td>
<td>8236.76</td>
<td>8237.123</td>
<td>8236.748</td>
</tr>
<tr>
<td>Total Q gen [MVAR]</td>
<td>2199.72</td>
<td>2199.128</td>
<td>2199.043</td>
</tr>
<tr>
<td>Elapsed Time [s]</td>
<td>0.95</td>
<td>0.582</td>
<td>2.874</td>
</tr>
</tbody>
</table>

Moreover, the modified model has introduced more variables which may cause the complexity of computation.

In summary, in case of using old method, the results are still accurate in a shorter execute time. However, the new method has increased the accuracy of simulation results with longer time. When the system becoming large or under high load operation, the effect of accuracy of voltage magnitude is getting larger when using the old method. The new method can reduce such effect, but the consuming time is much longer. There are advantages and drawbacks of both methods, according to different situation the more appropriate one should be chosen.
Table 3.9: OPF results from modified model (MM), SOCP OPF (SOCP) and MAT-POWER (MAT) of IEEE 2383-bus under normal operation

<table>
<thead>
<tr>
<th></th>
<th>MAT</th>
<th>SOCP</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost [$]</td>
<td>1.8784+e6</td>
<td>1.8775+e6</td>
<td>1.8782e+6</td>
</tr>
<tr>
<td>Total P losses [MW]</td>
<td>700.304</td>
<td>690</td>
<td>699.857</td>
</tr>
<tr>
<td>Total Q losses [Mvar]</td>
<td>3864.11</td>
<td>3840.6</td>
<td>3861.5</td>
</tr>
<tr>
<td>Total P gen [MW]</td>
<td>25258.7</td>
<td>25248.39</td>
<td>25258.3</td>
</tr>
<tr>
<td>Total Q gen [Mvar]</td>
<td>6675.2</td>
<td>6651.7</td>
<td>6672.6</td>
</tr>
<tr>
<td>Elapsed Time [s]</td>
<td>7.1</td>
<td>6.4</td>
<td>16.1</td>
</tr>
</tbody>
</table>
Chapter 4

Simulation results and comparison with PSS/E

In order to investigate the performance of SOCP formulation, we also compare the simulation results from SOCP formulation with the other commercial software. As is known to all, PSS/E is the best performing commercial software in the market, so PSS/E is used as the compared software.

In this chapter, a small test system (IEEE 30-bus) and a large test power system (IEEE 2383-bus) are both modeled with SOCP formulation using the GAMS MOSEK solver and in PSS/E OPF package. After the implementation of modeling with these two methods, the comparison of results is made.

4.1 PSS/E

PSS/E is widely used commercial software after it was introduced in 1976. This software is a set of computer programs which can deal with the basic function of simulation work of the power system. PSS/E can handle the simulation work as follows [25]:

- Power Flow
- Optimal Power Flow
- Fault Analysis
• Dynamic Simulations
• Open network Access and Price calculation
• Equivalent Construction

PSS/E is composed of some modules which can handle different simulations of small or large systems. In this project, we only use the power flow and optimal power modules of PSS/E.

PSS/E power flow module is the basis of PSS/E module and it can be used to simulate and analyze power flow network.

PSS/E optimal power flow (PSS/E OPF) module is an advanced PSS/E module. The OPF module can solve the power system with some constrains and limitations to achieve some economical goals. PSS/E OPF can achieve the optimization by minimizing different objective functions.

To solve the OPF problems in PSS/E OPF, the OPF model data must be set correctly. The OPF model data is consisted of transmission part and generation part of network.

The transmission part modeling in PSS/E can be implemented by inputting all the information of buses and connection links elements (branches, shunts, etc.). After that the network can be represented by the PSS/E model.

The generation part modeling in PSS/E is to model all generators respectively with proper data sets that can represent the characteristics of generators. The data sets are composed of three sections:

• Generation dispatching data
• Generation cost curve

The generation dispatching data is represented by using the dispatch tables in PSS/E. All the data in each table can describe the state of each generator in the network. The data sets are shown as follows:

• Maximum of generation Max
• Minimum of generation Min
Chapter 4. Simulation results and comparison with PSS/E

4.2 Simulation results

As can be seen from the simulation results from SOCP formulation and PSS/E, the results for IEEE 30-bus from SOCP formulation and PSS/E are almost identical, but the elapsed time of SOCP formulation coded in platform GAMS is longer than that of PSS/E. But when the SOCP formulation is executed in the GAMS software, it takes time to compile the data files and execute the commands, which account for significant portion of the elapsed time. The execute time of MOSEK optimizer is only 0.06 seconds. Figure 4.2 shows the power flow diagram of IEEE 30-bus after implementation modelling in PSS/E.
Figure 4.2: IEEE 30-bus power flow diagram
Chapter 4. *Simulation results and comparison with PSS/E*

For large system IEEE 2383-bus system, the results from SOCP formulation are also close to the results from PSS/E, the execute time of MOSEK optimizer is 0.63 seconds which is longer than the elapsed time of PSS/E, which is a little longer that the execute time of PSS/E. Compared with PSS/E OPF, the SOCP formulation still get high accuracy and efficiency for both small and large test systems.

Table 4.1: OPF results from SOCP OPF (SOCP) and PSS/E OPF of IEEE 30-bus (A)

<table>
<thead>
<tr>
<th>Gen</th>
<th>$P_G$ [MW]</th>
<th>$Q_G$ [MV AR]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSS/E</td>
<td>SOCP</td>
</tr>
<tr>
<td>1</td>
<td>63.19</td>
<td>63.18</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>27</td>
<td>0</td>
<td>0</td>
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Table 4.2: OPF results from SOCP OPF (SOCP) and PSS/E OPF of IEEE 30-bus (B)

<table>
<thead>
<tr>
<th></th>
<th>PSS/E</th>
<th>SOCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost [$]</td>
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<td>316.39</td>
</tr>
<tr>
<td>Total P losses [MW]</td>
<td>3.992</td>
<td>3.988</td>
</tr>
<tr>
<td>Total Q losses [Mvar]</td>
<td>15.057</td>
<td>15.051</td>
</tr>
<tr>
<td>Total P gen [MW]</td>
<td>193.19</td>
<td>193.188</td>
</tr>
<tr>
<td>Total Q gen [Mvar]</td>
<td>80.07</td>
<td>80.059</td>
</tr>
<tr>
<td>Elapsed Time [s]</td>
<td>0.065</td>
<td>0.526</td>
</tr>
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Table 4.3: OPF results from SOCP OPF (SOCP) and PSS/E OPF of IEEE 2383-bus

<table>
<thead>
<tr>
<th></th>
<th>PSS/E</th>
<th>SOCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost [$]</td>
<td>1.8785e+6</td>
<td>1.8782e+6</td>
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<tr>
<td>Total P losses [MW]</td>
<td>700.7</td>
<td>690</td>
</tr>
<tr>
<td>Total Q losses [Mvar]</td>
<td>3864.1</td>
<td>3840.6</td>
</tr>
<tr>
<td>Total P gen [MW]</td>
<td>25258.7</td>
<td>25248.9</td>
</tr>
<tr>
<td>Total Q gen [Mvar]</td>
<td>6675.1</td>
<td>6651.7</td>
</tr>
<tr>
<td>Elapsed Time [s]</td>
<td>0.336</td>
<td>6.409</td>
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</table>
Chapter 5

Conclusion and further work

5.1 Conclusion

This thesis has studied the developed second order cone programming formulation for OPF. The accuracy and computation time are considered as the two key aspects of studying.

The developed conic formulation of OPF is introduced in details firstly, approximated method and conic relaxation are used to transform constrains to second order programming format.

A modified model is developed to study on accuracy of the SOCP formulation and tested on four test systems including small and large systems under normal load and high load operation.

The modified model is also based on the line flow equations, only the phase angle constrains are modified to obtain a more accurate result. The differences of phase angle are described as functions of line flow variables and linearized in a Taylor’s series. The modified model can be solved by using sequential conic programming method.

The modified model has improved the accuracy of solution results. Especially, For large system or system under high load operation, the accuracy of voltage magnitude is improved a lot compared with the SOCP formulation. But the sequential method applied in modified model causes the algorithm complicated and slow speed of computation.
A comparison is carried out between the results of SOCP formulation and PSS/E software to investigate the performance of SOCP formulation. After implementation the SOCP formulation and PSS/E modelling on one small system and one large scale system, it can be found that the SOCP formulation is quite accurate even for larger system and the elapsed time is more than that of PSS/E, but it is because the time of compiling data and executing commands in GAMs platform, for the small system the MOSEK optimizer time is less than the elapsed time of PSS/E.

After studying the performances of SOCP formulation for OPF problem, we can say that the SOCP formulation of OPF can be seen as an accurate and reliable formulation for OPF problems and can be solved efficiently by using IPMs.

5.2 Further work

In this project, we only consider minimizing the total fuel cost as the objective function, some other objective functions such as minimizing the active losses can be studied to analyze the performances of SOCP formulation. Since the application of FACTS in the power systems is increasing, to model the FACTS in the power systems in the SOCP formulation is an important work for future.

The fundamental loops are not used in modified model, the modified model only use linearized equations to represent the differences of phase angles, the modified model can be used to analyze the optimal transmission switching problems.
Bibliography


